Enhanced Transport Caused by Magnetic Field Perturbations in Field Reversed Configurations*

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ABSTRACT

As has recently been shown [1], magnetic field perturbations have a strong effect on the magnetic field structure of “classical” FRCs, the ones where only poloidal magnetic field is present. In particular, even very small perturbations can, in principle, lead to large radial excursions of the field lines [1]. The particle motion in this case is a competition between the (toroidal) curvature drift across the field lines and the radial excursions due to streaming along the field lines. Introduction of a weak regular toroidal magnetic field reduces radial excursions of the field lines. Possible source of perturbations could be low-frequency non-MHD modes in plasmas with a large enough beta values, as well as imperfections of the coils. Neoclassical transport arises when particle collisions are included into analysis. We estimate transport coefficients in various collisionality regimes and formulate constraints on admissible field errors. Work performed for U.S. DoE by UC LLNL under contract W-7405-ENG-48.

OUTLINE

• Motivation

• Characterization of perturbed magnetic field structure

• Electron losses from FRC

• Introducing toroidal magnetic field (to reduce electron losses)

• Collisional ion losses in the presence of the toroidal field

• Further issues

• Conclusion
Motivation

There exist confinement systems where only poloidal magnetic field is present: FRC, levitated dipole, long diffuse pinch

Effect of magnetic perturbations on the magnetic field structure in such systems cannot be described in terms of the familiar island formation (and possible overlapping of the islands) as is the case in the systems with a substantial toroidal field, like tokamaks, RFPs, spheromaks.

It turns out that the magnetic field structure in the “poloidal field only” systems is very sensitive to perturbations.

Magnetic field perturbations may lead to enhanced transport.

We use coordinates $\Phi$ (poloidal flux of the unperturbed field), $l$ (length of the unperturbed field line measured from the equatorial plane), $\varphi$ (toroidal angle).

We decompose magnetic field perturbations over three mutually perpendicular directions: the direction normal to the flux surface ("radial"), the direction tangential to the flux surface and lying in the poloidal plane, and the toroidal direction. We denote these components by $\delta B_n$, $\delta B_p$, and $\delta B_t$, respectively.
For small perturbations,
\[
\frac{d\Phi}{dl} = 2\pi R \delta B_n ; \quad \frac{d\varphi}{dl} = \frac{\delta B_t}{B_0 R}
\]
The (small) variation of $\Phi$ and $\varphi$ during one full turn in the poloidal direction is:
\[
\Delta \Phi = 2\pi \oint R \delta B_n \, dl; \quad \Delta \varphi = \oint \frac{\delta B_t \, dl}{RB_0}
\]
Right-hand sides of these equations are functions of $\Phi$ and $\varphi$.
\[
\frac{d\Phi}{d\varphi} = \frac{2\pi \oint R \delta B_n \, dl}{\oint \frac{\delta B_t \, dl}{RB_0}} \equiv F(\Phi, \varphi)
\]
This result is quite general in that it is not based on any assumptions about the plasma beta. It is equally applicable to the levitated dipoles, FRCs, and diffuse pinches.
A remarkable feature of the “poloidal-field-only” systems: even an infinitesimal perturbation causes a dramatic change of the magnetic topology: without perturbations, each field line would have generated only a point on the \((r, \varphi)\) plane, determined by the initial \(\varphi\) and \(r\).

\[
\frac{d\Phi}{d\varphi} = \frac{2\pi \oint R \delta B_n \, dl}{\oint \frac{\delta B_t \, dl}{RB_0}}
\]
The path $s$ (the number of turns around the field null) that particular field line makes before getting displaced by a substantial distance in the radial direction is, of course, determined by the amplitude of the perturbation. For the field line to be displaced radially by characteristic size $a$ (of order of a radial length-scale of perturbations), the field line has to make a path

$$ s \sim aB/\delta B $$

The “thickness” of the bananas is determined by the structure of the perturbation, not by its amplitude (!).

The FRC axis (axis $z$) is perpendicular to the plane of the figure; $xy$ is the equatorial plane. Shown in red is the line of a zero magnetic field (of the unperturbed FRC).
One more illustration: perturbation of the form of a uniform magnetic field perpendicular to the axis of the device and directed along the axis $x$:

$$\delta B_n = b \cos \varphi; \quad \delta B_t = -b \sin \varphi$$

A racetrack-shaped FRC ($L \gg a$)  

Puncture plots in the equatorial plane
Selection rules: radial wandering of the field line is zero, if

- The magnetic field perturbation is of an ideal MHD type, $\delta B = \nabla \times \xi \times B$

- The perturbation is axisymmetric, $\partial/\partial \phi = 0$

- The perturbation is anti-symmetric with respect to the equatorial plane $\delta B_n(z) = -\delta B_n(-z)$ (S.A. Cohen, R.D. Milroy. Phys Plasmas, 7, 2539, 2000)

An interesting point: only untrapped particles experience large radial wandering

$$\Delta \Phi = 2\pi \int R \delta B_n \, dl; \quad \Delta \varphi = \int \frac{\delta B_t \, dl}{RB_0}$$
Are the effects discussed here significant?

A numerical example: a system with the separatrix radius $a \sim 1 \text{ m}$ designed for a confinement of a plasma with electron temperature $T_e \sim 20$ keV, exposed to a perturbation with a relative amplitude

$$\varepsilon \sim \frac{\delta B}{B_0} \sim 10^{-2}$$

At this amplitude, the length $s$ of the field line before it gets displaced radially by the distance $\sim a$ is $\sim 100 \text{ m}$. For the 20-keV electron, the transit time over such a distance is a mere $1 \mu\text{s}$, many orders of magnitude less than the required confinement time (for typical FRC densities of $\sim 10^{16} \text{ cm}^{-3}$).

Everywhere in this paper we consider global-scale perturbations (small mode numbers)
Additional effects influencing electron transport:

- **Azimuthal drift**

\[ v_D \sim v_T (\rho/a); \quad \tau_D \sim a/v_D; \quad \tau_\parallel \sim s/v_T \sim (a/v_T)(B/\delta B) \]

Azimuthal drift is important if \( \tau_D < \tau_\parallel \). For \( \delta B/B \sim 10^{-2} \) never important for electrons; may be important for the ions in the MTF environment

- **Collisions**

Important if \( \lambda < s \sim a(B/\delta B) \); can be important in the MTF environment
Plasma parameters in several versions of FRC

We assume that in both cases $\delta B/B \sim 10^{-2}$, and the perturbation is of a “global” nature.

|       | $n$, cm$^{-3}$ | $T$, keV | $B$, T | $a$ (s) cm | $\lambda$, cm | $\rho_e$ cm | $\tau_{||e}$ ($\tau_{||i}$), s | $\tau_D$, s |
|-------|----------------|----------|--------|-------------|---------------|-------------|-----------------------------|-------------|
| Standard | $10^{16}$ | 20       | 10     | $100$ ($10^4$) | $10^5$         | $4 \cdot 10^{-3}$ | $10^{-6}$ $(6 \cdot 10^{-5})$ | $3 \cdot 10^{-4}$ |
| High-density | $10^{21}$ | 10      | 1000   | 0.1 ($10$)   | 0.3           | $3 \cdot 10^{-5}$ | $2 \cdot 10^{-9}$ $(10^{-7})$ | $5 \cdot 10^{-8}$ |
| FRX       | $10^{17}$ | 0.15     | 3      | 3 ($300$)    | 0.6           | $10^{-3}$     | $5 \cdot 10^{-7}$ $(3 \cdot 10^{-5})$ | $10^{-5}$   |

Electrons experience free-streaming losses in the case of a classical FRC (very fast!) and diffusive ($\chi_{||}$) losses in the case of short-mean-free-path plasmas.

Free streaming: 

$$\tau_{E}^{(e)} \sim \frac{B}{\delta B} \frac{a}{\nu_{Te}}$$

Diffusive: 

$$\tau_{E}^{(e)} \sim \left(\frac{B}{\delta B}\right)^2 \frac{a^2}{\lambda \nu_{Te}}$$

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To suppress electron heat losses, one can introduce a toroidal magnetic field, $B_t$, an order of magnitude higher than $\delta B$ (but still smaller than $B_p$)

$$\frac{d\Phi}{d\varphi} = \frac{2\pi \int R \delta B_n dl}{\int B_t dl / RB_0}$$

In this case, the radial wandering of field lines becomes of order of

$$\delta r \sim a \frac{\delta B}{B_t} \ll a$$

Most importantly, the “bananas” and open-field-line regions disappear.
The presence of the toroidal field leads to a new mechanism of the ion losses.

We consider it for the weakly-collisional “classical” FRC reactor parameters.

The ion toroidal velocity:

\[ v_t = \frac{B_t}{B} v_\parallel + v_D; \quad v_D \sim v_{Ti} \frac{\rho_i}{a} \sim \frac{v_{Ti}}{300} \]

At \( v_\parallel = -v_D (B/B_t) \sim v_{Ti} (\rho/a) (B/B_t) \sim v_{Ti} / 30 \), one has \( v_t = 0 \).

These ions are almost stagnant near the equatorial plane and experience strong effect of local perturbations. The interval of \( v_\parallel \) where this occurs is determined by the equation:

\[ \Delta v_\parallel \sim \min [v_{Ti} \delta B_r / B_t; v_D (B/B_t)] \]

The ions in this range of velocities are rapidly lost,

\[ \tau_{loss} \sim \frac{a}{v_{Ti}} \frac{B}{\delta B_n} \Lambda \]
The ion confinement time is determined by the collisional loss rate to this “prompt loss” zone:

\[ \tau_E^{(i)} \sim \tau_{ii} \]
Other effects to be considered:

Perturbations created by non-steady-state convection. If the characteristic correlation time is shorter than the time within which a particle makes a full radial excursion, the fluctuating nature of perturbations becomes important. This factor may both decrease the transport and increase it (the latter would occur for the electron whose drift velocity resonates with phase velocity of perturbations).

Fast particles loss (in particular, alpha particles). For fast particles, drift effect is substantial, and larger volume in the velocity space is subject to prompt losses.
Summary

- In FRC (as well as in other systems where only poloidal field is present), large radial excursions of magnetic field lines may occur even at very small perturbations

- This effect may lead to a very fast electron energy loss

- If the fluctuations are not too high, $\delta B/B < 0.03$, electron losses can be reduced by introducing moderate toroidal magnetic field ($\sim B/10$)

- Particle drifts and particle collisions lead to neoclassical-like ion diffusion which, in some cases, can be large

- Unexplored issues include the role of temporal dependence of fluctuations, and behavior of fast particles