Waves in Plasmas II:
Parametric Instabilities and Nonlinearities
(Laser-plasma instabilities)

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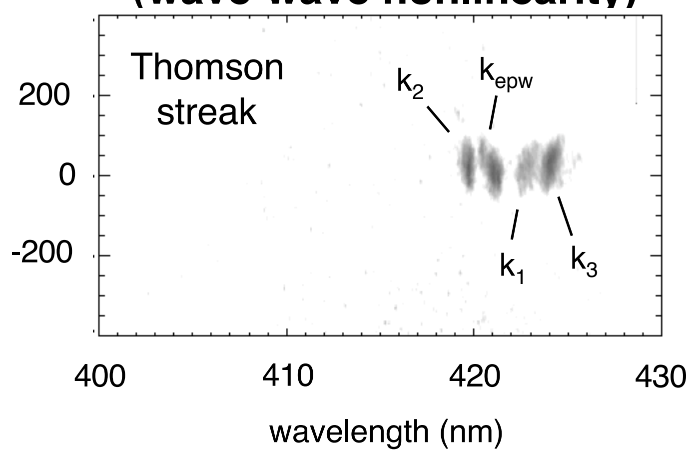
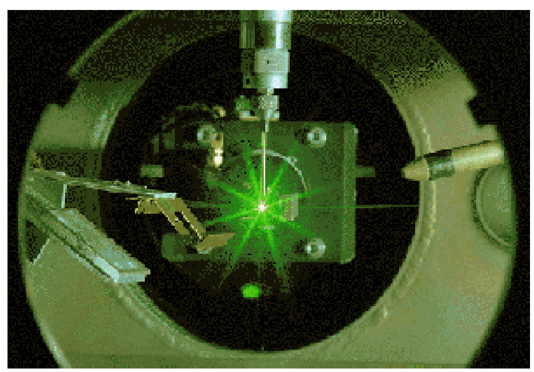
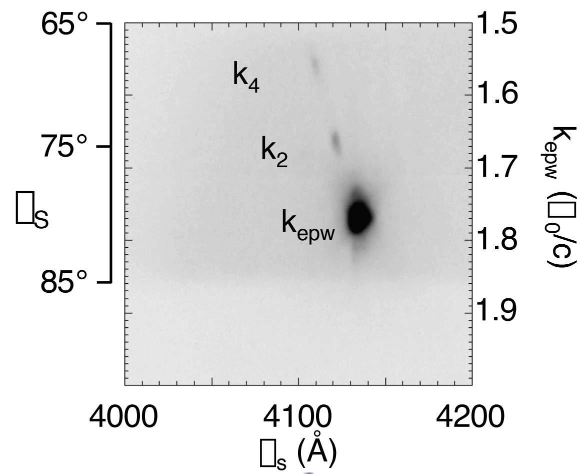
Langmuir Decay Instability cascades (wave-wave nonlinearity)



Los Alamos Trident Laser Facility



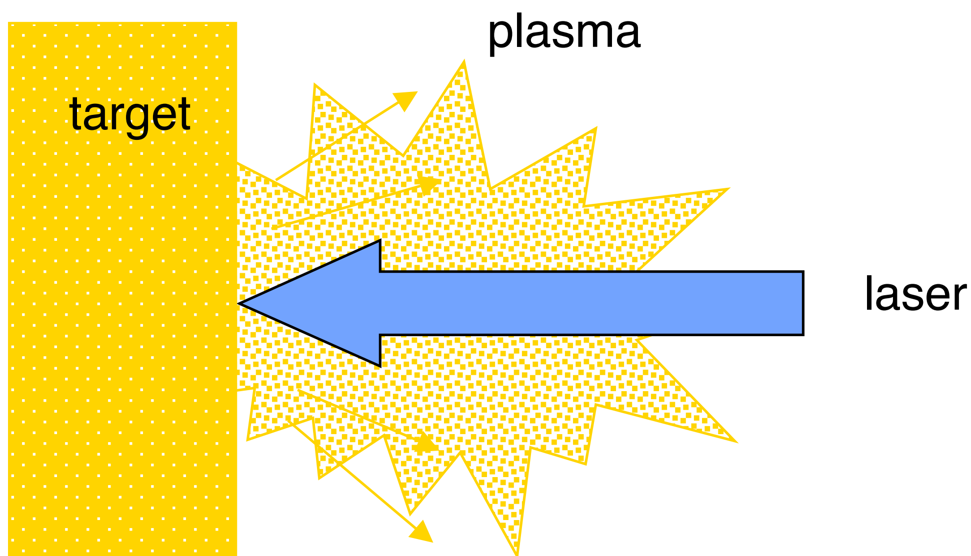
LDI (□, k) spectrum



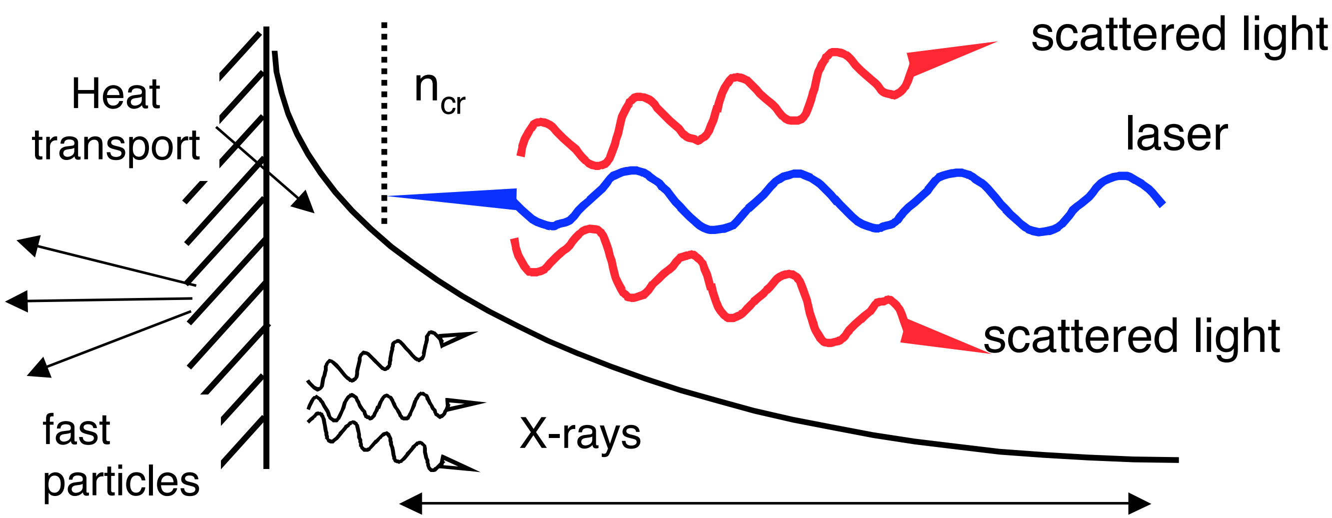
Overview of talk

- Waves in plasmas II: parametric instabilities & nonlinear phenomena
- Motivation: laser-plasma applications and laser fusion
- Revisit oscillations and waves in plasmas
- Coupled oscillators: parametric instability
- Some types of parametric instability important to laser fusion
- Particle trapping nonlinearities
- Examples from experiments
- Other types of nonlinear phenomena
- Summary

Laser Plasma Interaction: *extreme physics*



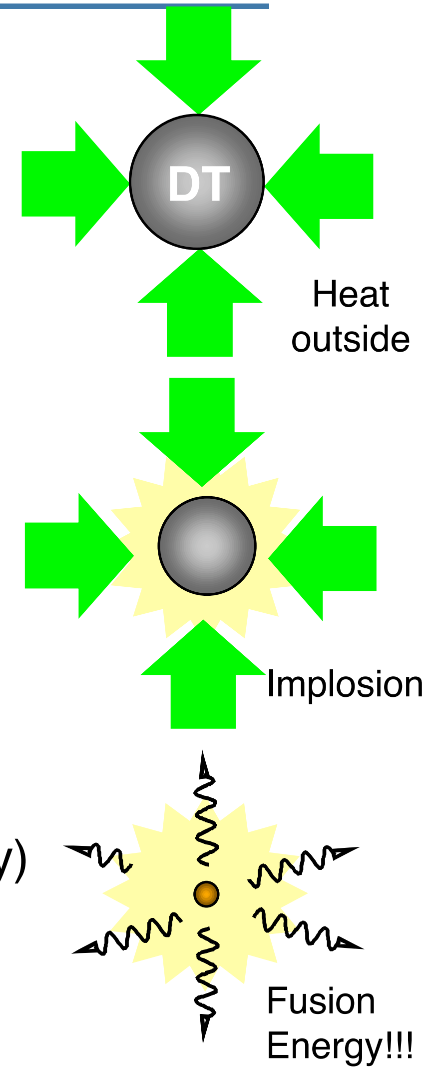
Laser Intensity: 10^{12} - 10^{20} W/cm^2
(30,000 GV/m)
Pulse Widths: 10^{-8} - 10^{-14} sec
Temperatures: 10^6 - 10^9 °C
Plasma flows: 10^7 - 10^8 cm/sec
(100's mile/sec)
Plasma size: 1 μm - 1 mm



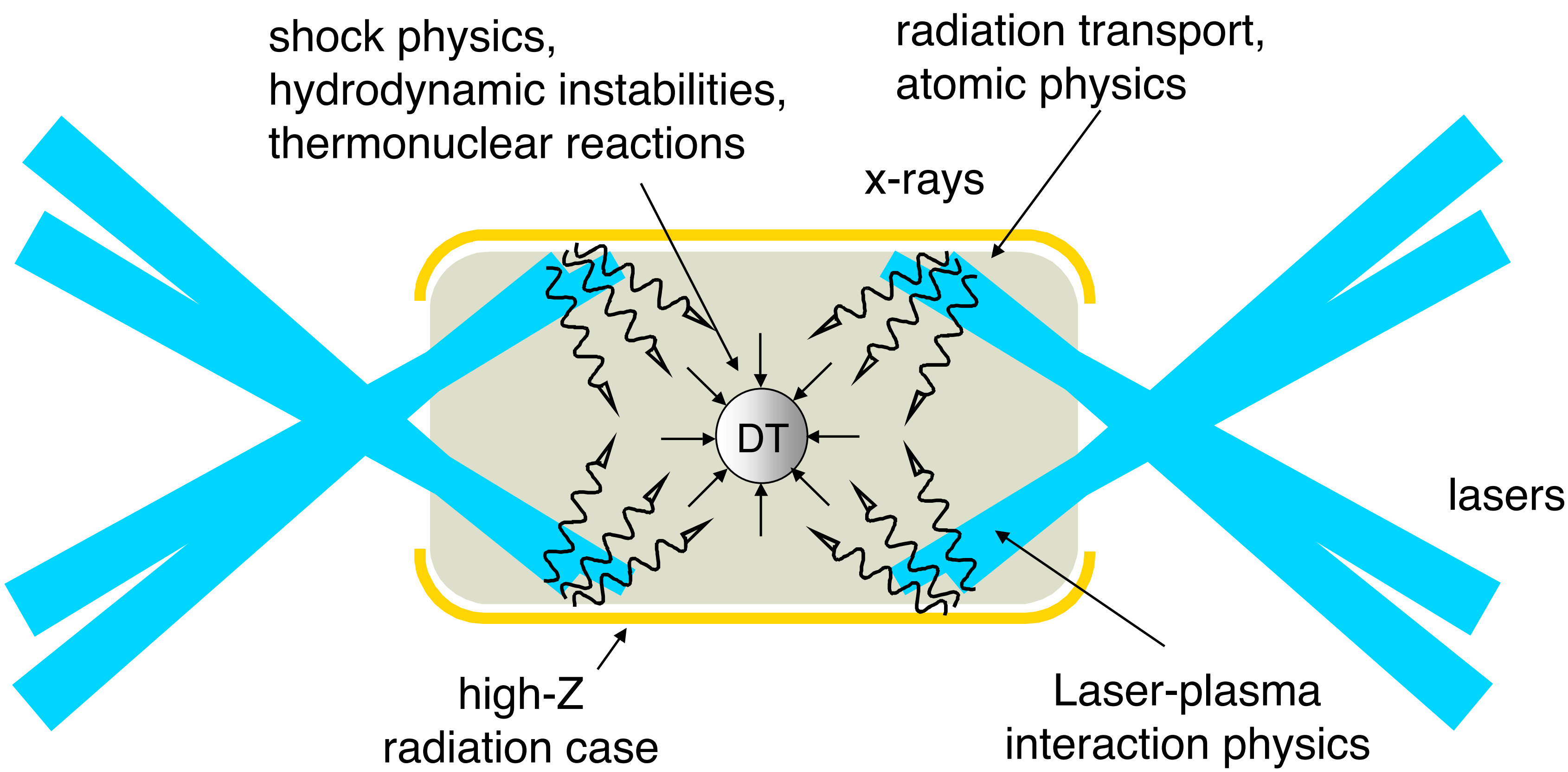
- Inverse Bremsstrahlung Absorption (e-i collisions)
- Parametric Instabilities

Several important laser-plasma applications have been developed over the years

- Inertial Confinement Fusion (laser fusion)
- X-ray lasers
- Plasma-based accelerators
- Novel, ultrashort laser pulse amplification
- VUV and X-ray lithography
- Plasma processing
- Remote sensing (laser-induced breakdown spectroscopy)
- Ion, proton beam production
- Basic and applied physics research
  - direct nuclear excitation
  - high energy-density physics (high T_e, pressure, density)
  - laboratory astrophysics
  - laboratory general relativity simulations?
  - others...

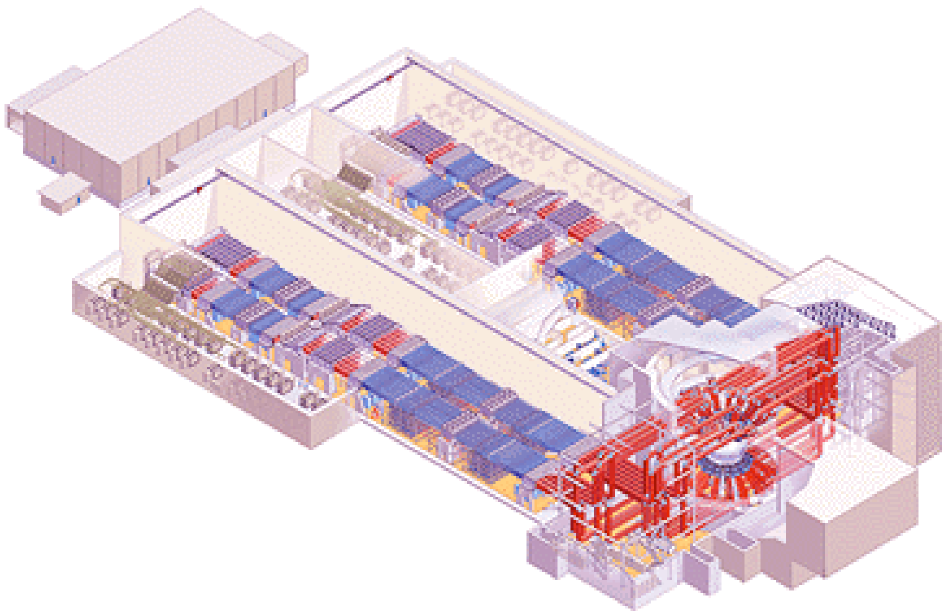


Inertial confinement fusion is a grand challenge, requiring a multi-disciplined approach



- high power laser technology
- high-resolution, ultra-high-speed diagnostics
- material science, precision micro-fabrication
- advanced computing

The National Ignition Facility (NIF) is expected to achieve thermonuclear ignition



- 1.8 MJ in 20 billionths of a second from 192 beams of 351 nm light

- Designed for 20 MJ of fusion output

The largest laser facility in the world!

- target experiments began in 2003 with 1st 4 beams

- all 192 beams to be completed by 2008

Major LPI physics and ICF research institutions worldwide

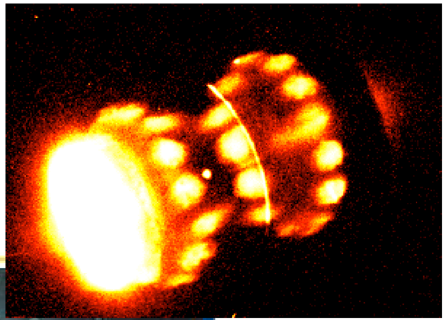
North America: LLNL, LANL, Univ. Rochester, NRL, Sandia, UCLA, GA, UBC, Univ. Alberta, NRC (Canada)

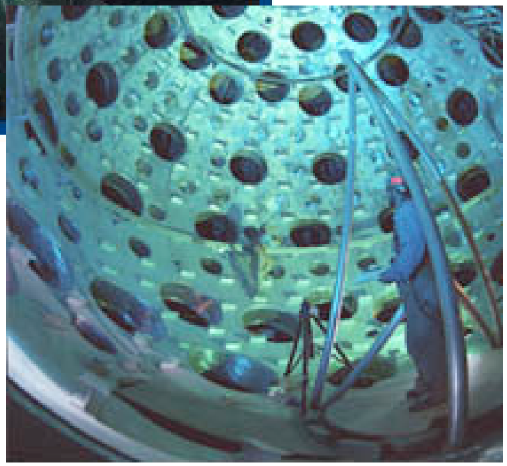
UK: AWE, Imperial College, RAL, Oxford

Japan: ILE Osaka

France: CEA, Ecole Polytechnique

Germany, Czech Rep., Italy, Spain, China, Russia,…





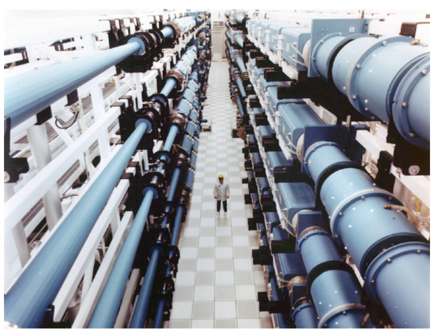
Major U.S. high-power laser facilities

Trident
(Los Alamos)
500 J

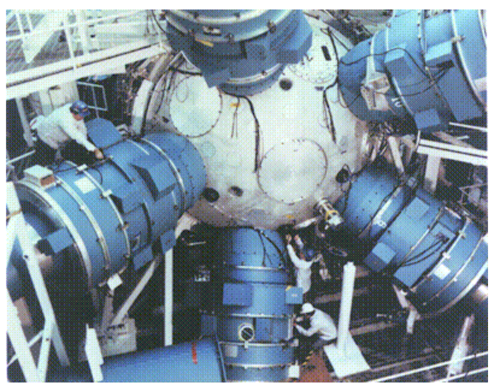
Omega Laser
(Univ. of Rochester)
30 kJ, 30 TW

Z Backlighter
(Sandia, Albuquerque)
10 kJ, 10 TW

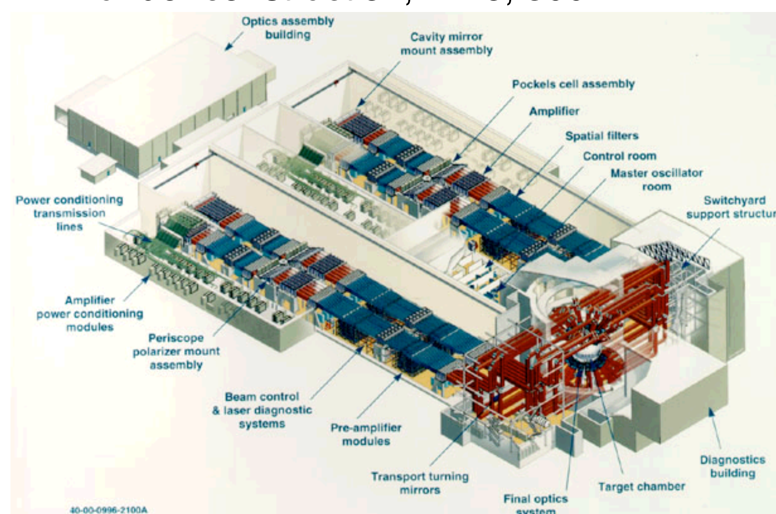
Nova Laser
(Livermore)
Defunct
30 kJ, 30 TW



Nova
Target chamber
(defunct)



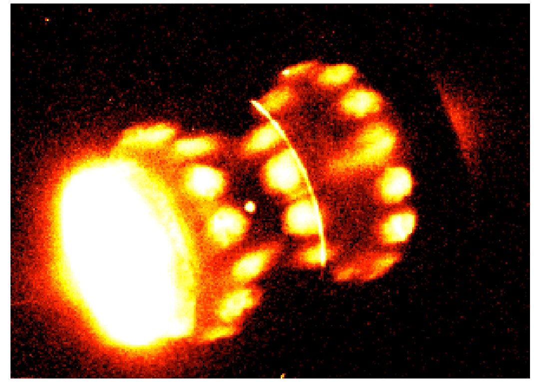
NIF - National Ignition Facility
(Livermore)
under construction, 2 MJ, 500 TW



Cool Laser Fusion Pictures

Inside Omega chamber
during a fusion experiment

X-ray micrograph of Hohlraum
experiment on Omega

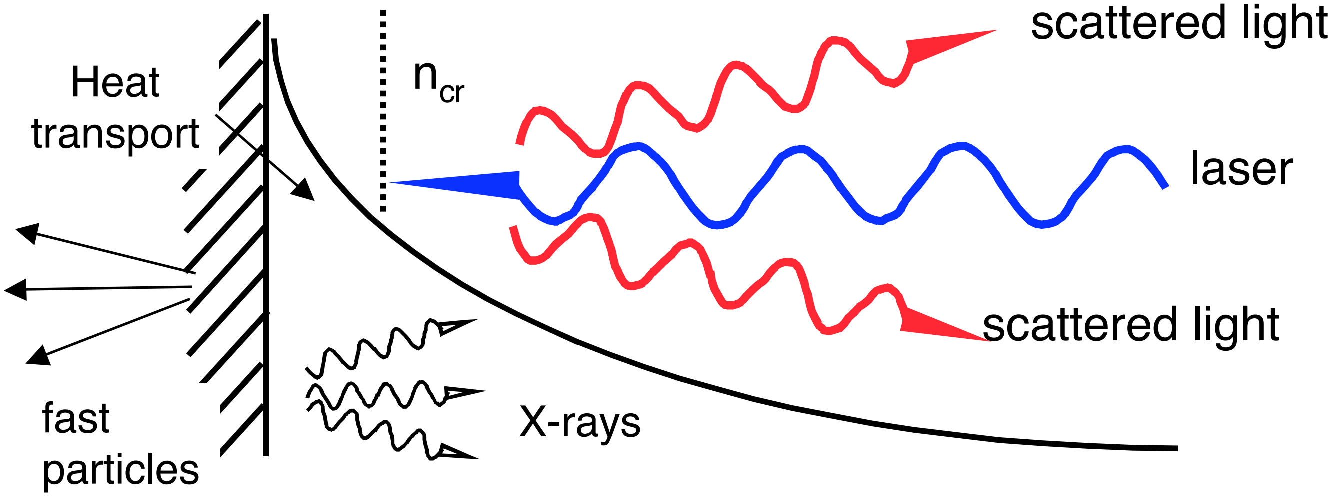


NIF target chamber
(under construction)

30 ft. dia.
1 million lbs.

Fusion Yield (DT) ≈ 10^{14} (14.1 MeV) neutrons
~ 300 J of fusion energy
~ 30 kJ of laser energy
~ 1% fusion yield

Ideally, one wants to gently heat the plasma by collisional absorption of the intense laser beam

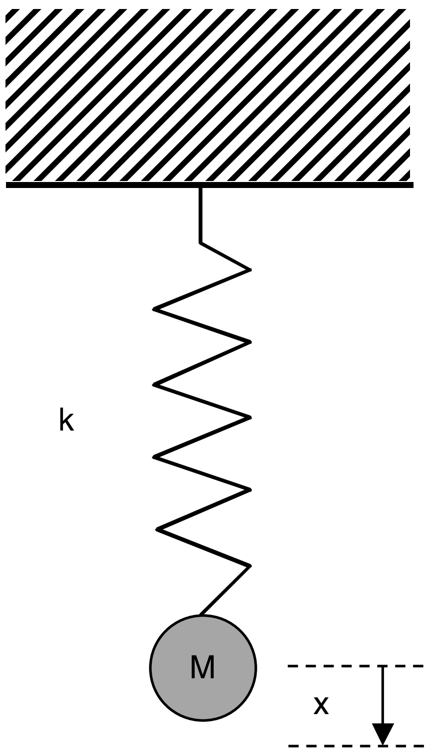


- collisional absorption (inverse bremsstrahlung)
  - electrons oscillating in laser field collide with ions, transferring energy from the laser to the electrons

- n_{cr} where $\omega_0 = \omega_p = (4\pi n e^2/m_e)^{1/2}$, electrons short laser E-field

- however, parametric instabilities create undesirable effects:
  - scattered light (energy loss, redirection of light, possible laser damage)
  - energetic particles (preheat fusion capsule)

Plasma oscillations have a simple analogy to a mass on a spring



mass **M** suspended on a spring of constant κ
(spring provides restoring force $F = \kappa x$)

$$F = Ma = -\kappa x$$

$$M\frac{d^2x}{dt^2} + \kappa x = 0$$

$$\frac{d^2x}{dt^2} + \frac{\kappa}{M}x = 0 \qquad \text{Eq. of motion}$$

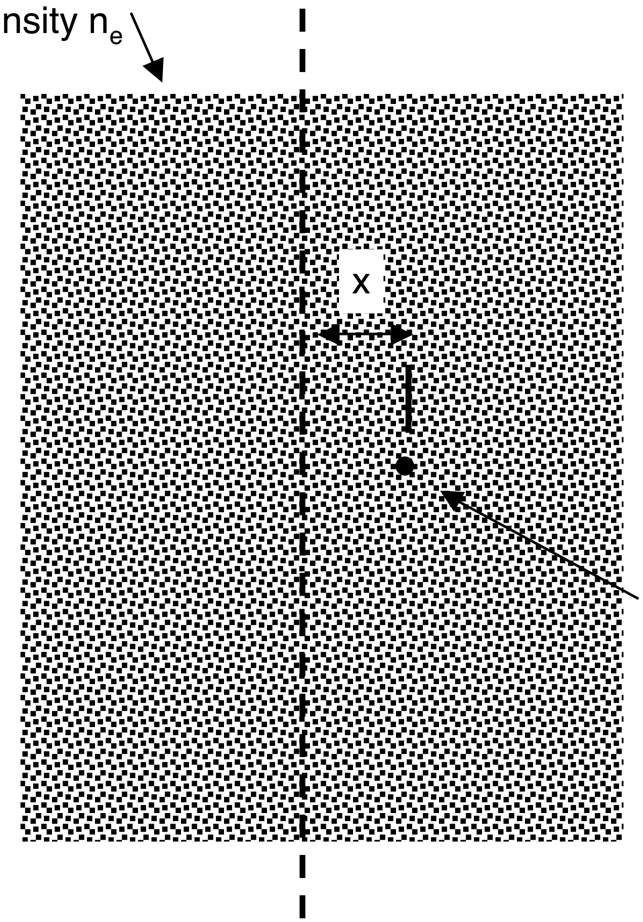
$$\frac{d^2x}{dt^2} + \omega^2 x = 0, \quad \omega = \sqrt{\frac{\kappa}{M}}$$

$x(t) = A \sin\omega t + B \cos\omega t$

$x(t) = A_0\exp(-i\omega t)$

mass oscillates with frequency ω

Electron Plasma Oscillations: a simple derivation



Electric field is
restoring force

$$m_e \frac{d^2x}{dt^2} = -eE \quad \text{(Force Eq.)}$$

$$\frac{dE}{dx} = 4\pi n_e e \quad \text{(Poisson's Eq.)}$$

$$m_e \frac{d^2x}{dt^2} + eE = 0$$

$$E(x) = 4\pi n_e e x$$

$$\frac{d^2x}{dt^2} + \frac{4\pi e^2 n_e}{m_e} x = 0 \quad \text{(Eq. of motion)}$$

$$\frac{d^2x}{dt^2} + \omega_p^2 x = 0, \quad \omega_p^2 \equiv 4\pi e^2 n_e / m_e$$

$$x(t) = A\exp(-i\omega_p t)$$

ω_p is the plasma frequency

Electron Plasma Wave Derivation: *advanced*

$$\frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0 \qquad (1) \qquad \text{Continuity Eq.}$$

$$n\left(\frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial x}\right) = \frac{-e}{m_e} nE - \frac{1}{m_e}\frac{\partial P}{\partial x} \qquad (2) \qquad \text{Force Eq.}$$

Goal: develop wave equation for density fluctuations.
Assumptions: B=0, small amplitude ∂n, fixed ions, adiabatic

since $\omega/k >> v_{th}$, 1-D adiabatic $\frac{\partial P}{\partial x} = \gamma k_B T \frac{\partial n}{\partial x}$, $\gamma = \frac{2+N}{N} = 3$

taking $\partial/\partial t$ of Eq.1, $\partial/\partial x$ of Eq. 2, and subtracting to cancel out $\partial(nv)/\partial x \partial t$ terms, we have

$$\frac{\partial^2 n}{\partial t^2} - \frac{\partial\left(nv^2\right)}{\partial x^2} - \frac{e}{m_e}\frac{\partial (nE)}{\partial x} - 3v_{th}^2 \frac{\partial^2 n}{\partial x^2} = 0, \quad v_{th}^2 \equiv \frac{k_B T}{m_e}$$

next, linearize by $n = n_0 + \tilde{n}$, $E = \tilde{E}$, $v = \tilde{v}$, and assume steady-state, homogeneous $\left(\partial n_0/\partial t = \partial n_0/\partial x = 0\right)$, products of $\tilde{E}$, $\tilde{n}$, $\tilde{v} \to 0$

$$\frac{\partial^2 \tilde{n}}{\partial t^2} - \frac{e n_0}{m_e}\frac{\partial \tilde{E}}{\partial x} - 3v_{th}^2 \frac{\partial^2 \tilde{n}}{\partial x^2} = 0$$

Poisson's Eq. $\frac{\partial \tilde{E}}{\partial x} = -4\pi e \tilde{n}$

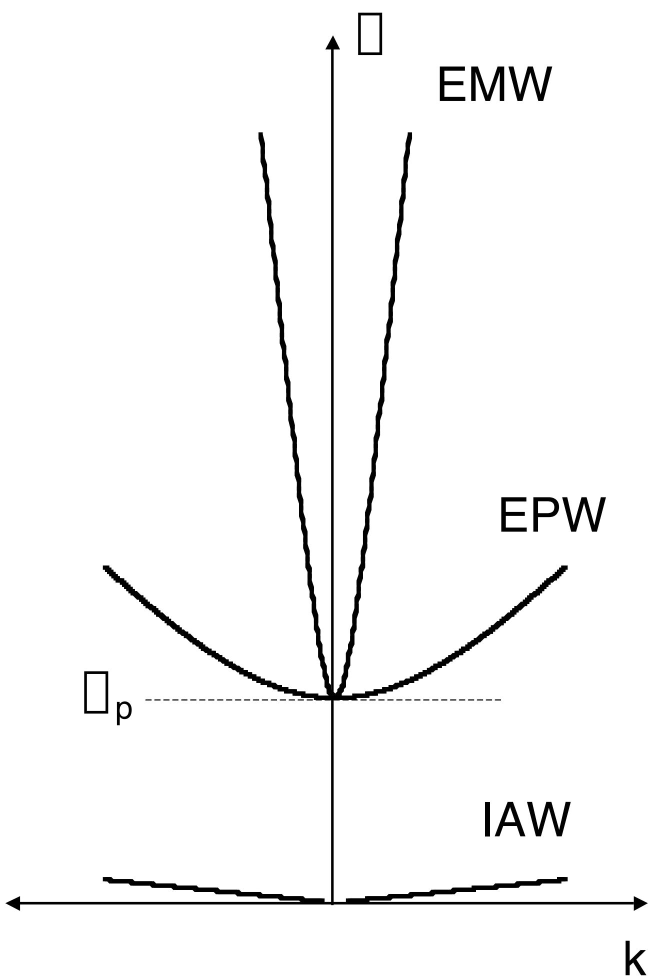
$$\frac{\partial^2 \tilde{n}}{\partial t^2} + \frac{4\pi e^2 n_0}{m_e}\tilde{n} - 3v_{th}^2 \frac{\partial^2 \tilde{n}}{\partial x^2} = 0$$

$$\left(\frac{\partial^2}{\partial t^2} + \omega_p^2 - 3v_{th}^2 \frac{\partial^2}{\partial x^2}\right)\tilde{n} = 0$$

looking for wave-like solutions $\tilde{n} \sim e^{i(kx-\omega t)}$, Fourier-analyzing

$$\omega^2 = \omega_p^2 + 3k^2 v_{th}^2$$

An unmagnetized plasma supports three natural modes of oscillation (waves)



EMW

$$\omega_0^2 = \omega_p^2 + k_0^2 c^2$$

EPW or Langmuir

$$\omega_{epw}^2 \approx \omega_p^2 + 3k_{epw}^2 v_e^2 \approx \omega_p^2\left(1 + 3k_{epw}^2 \lambda_D^2\right)$$

IAW

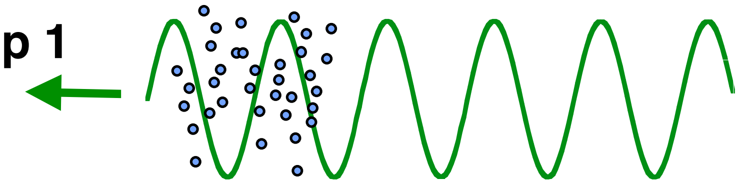
$$\omega_{iaw} \approx c_s k_{iaw} + \vec{k}_{iaw} \cdot \vec{v}$$

$$\omega_p = \left(4\pi n e^2 / m_e\right)^{1/2},\ v_e = \left(k_B T_e / m_e\right)^{1/2},\ \lambda_D = v_e / \omega_p$$

$$c_s \approx \left(Z k_B T_e / M_i\right)^{1/2}$$

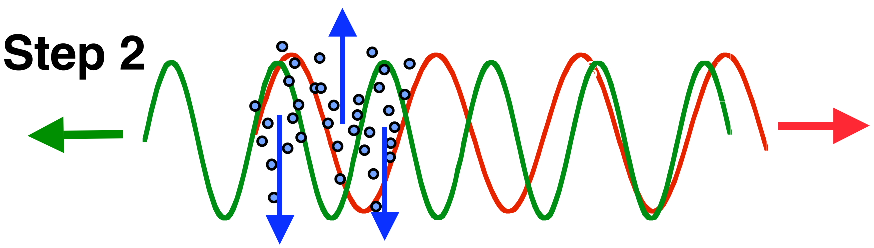
How does a 3-wave parametric instability grow in a plasma?

Step 1



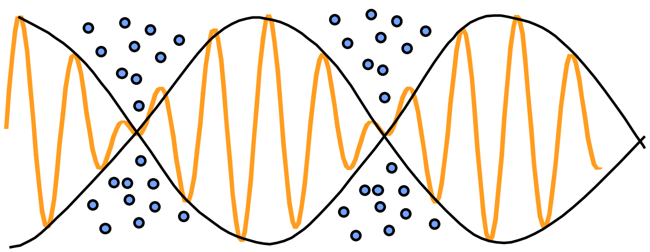
Laser light propagates through the plasma

Step 2



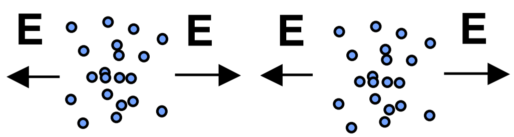
Oscillations in the plasma begin to radiate scattered light

Step 3



The beating of the two light waves creates a ponderomotive force pushing the particles into the troughs of the envelope

Step 4



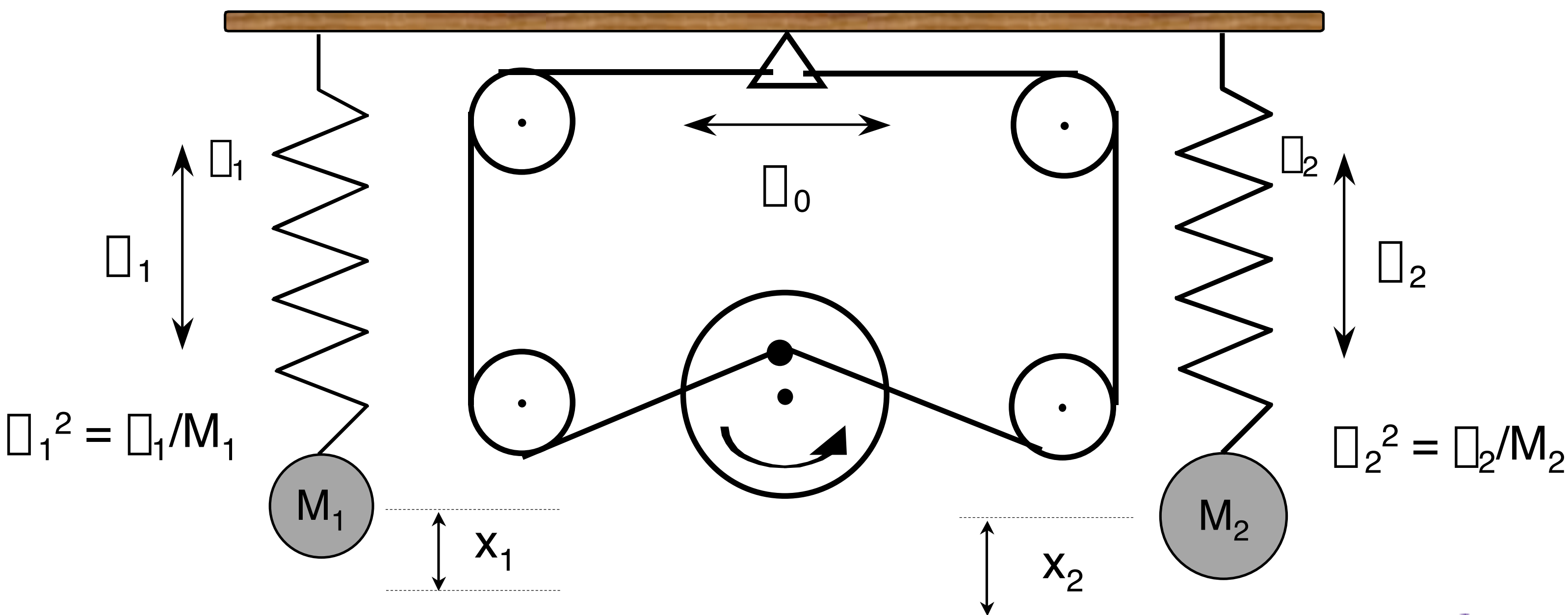
Electrostatic plasma wave

If the bunching of the particles matches an electrostatic mode, the 3 waves become resonant and grow

Parametric instability of coupled oscillators: a mechanical analogy (from Chen)

Resonance occurs when the “driving frequency” ω_0 equals the sum/difference of the two “daughter” oscillation frequencies:

$\omega_0 = \omega_2 \pm \omega_1$,   oscillation amplitude exponentiates!!



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Stability analysis for coupled oscillators

$$\frac{d^2x_1}{dt^2} + \omega_1^2 x_1 = 0, \; undriven$$

$$\frac{d^2x_1}{dt^2} + \omega_1^2 x_1 = c_1 x_2 E_0$$

$$\frac{d^2x_2}{dt^2} + \omega_2^2 x_2 = c_2 x_1 E_0$$

$$let\; x_1 = x_1 \cos\omega t,\; x_2 = x_2 \cos\omega' t,\; E_0 = E_0 \cos\omega_0 t$$

$$\begin{vmatrix} \omega^2 - \omega_1^2 & c_1E_0 & c_1E_0 \\ c_2E_0 & (\omega_0 - \omega)^2 - \omega_2^2 & 0 \\ c_2E_0 & 0 & (\omega_0 + \omega)^2 - \omega_2^2 \end{vmatrix} = 0$$

$\omega = \omega_1 + i\gamma$, unstable for $\gamma > 0$, damped $\gamma < 0$

amplitudes x_1, x_2 grow when $\omega_0 \approx \omega_2 \pm \omega_1$

when $\gamma << \omega$

$$\gamma = \frac{|E_0|\sqrt{c_1c_2}}{\sqrt{16\omega_1\omega_2}}$$, temporal growth rate $[x_1, x_2 \sim \exp(\gamma t)]$

oscillators ω_1, ω_2 have damping Γ_1, Γ_2

$$\frac{d^2x_1}{dt^2} + \omega_1^2 x_1 + 2\Gamma_1 \frac{dx_1}{dt} = 0$$

$\vdots$

Threshold condition (for $\omega_0 = \omega_1 + \omega_2$)

$$\gamma^2 \geq \Gamma_1\Gamma_2$$

When the coupled oscillators are waves:

$$\exp(-i\omega t) \rightarrow \exp[i(\vec{k}\cdot\vec{r} - \omega t)]$$

wave matching (resonance) conditions:

$$\omega_0 = \omega_1 + \omega_2$$

$$\vec{k}_0 = \vec{k}_1 + \vec{k}_2$$

$\hbar\omega_0 = \hbar\omega_1 + \hbar\omega_2$ (conservation of energy)

$\hbar\vec{k}_0 = \hbar\vec{k}_1 + \hbar\vec{k}_2$ (conservation of momentum)

A host of three-wave resonant decay processes are energetically possible in a laser-plasma

1) *EMW* → *EMW* + *EPW* (stimulated Raman scatter, SRS)

2) *EMW* → *EMW* + *IAW* (stimulated Brillouin scatter, SBS)

3) *EMW* → *EPW* + *IAW* (parametric decay instability)

4) *EMW* → *EPW* + *EPW* (two plasmon decay)

5) *EPW* → *EPW* + *IAW* (Langmuir decay instability, LDI)

6) *EPW* → *EMW* + *IAW* (electromagnetic decay instability)

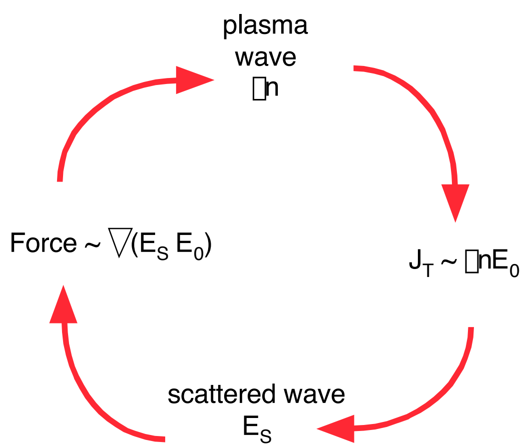
7) *IAW* → *IAW* + *IAW* (two ion wave decay)

other nonlinear processes:

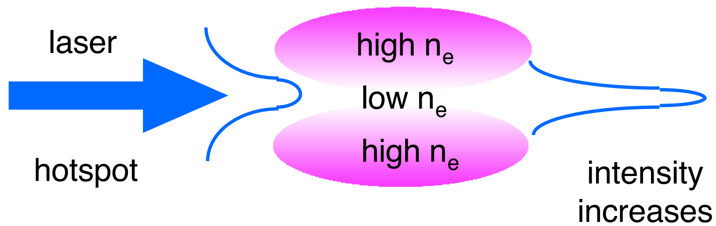
- modulational instabilities
- self focusing
- particle trapping
- mode coupling

Parametric coupling between intense laser wave, plasma wave and scattered light wave

Parametric Instability
Feedback loop



Self-focusing (filamentation)



Instability occurs when growth rate exceeds damping of wave energy in interaction volume

Interplay and competition can occur between these instabilities

Instabilities Remain a Threat to ICF

Nova large plasma experiments
Up to 50% SRS
Up to 30% SBS

$$\omega_0 = \omega_s + \omega_{es}$$
$$\vec{k}_0 = \vec{k}_s + \vec{k}_{es}$$

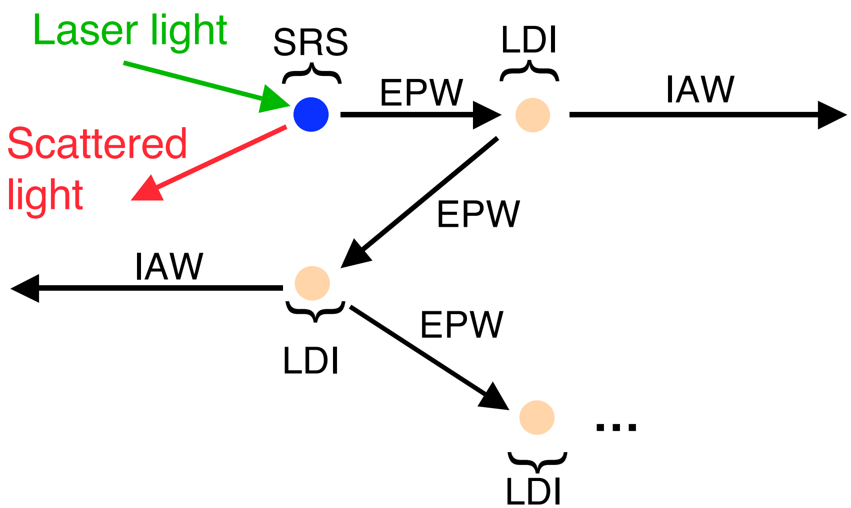
Stimulated Raman and Stimulated Brillouin scattering:
- energy loss
- scattered light in undesired locations
- preheat due to hot electrons (SRS) (killed Helios and Shiva)
- potential damage to NIF optics (SBS)

Self-focusing
- beam breakup, spraying, steering
- enhance SRS, SBS

Non-linear effects on Langmuir waves from Stimulated Raman Scattering (SRS) are sensitive to the ratio of the electron thermal speed to the wave phase speed, i.e. $k\lambda_D$

Wave-Wave ($k\lambda_D \sim v_e/v_\phi << 1$)

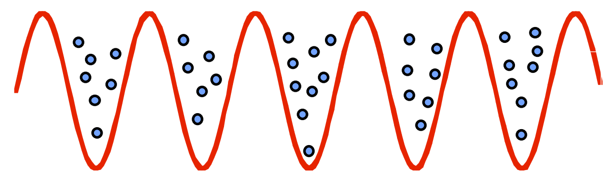
Langmuir Decay Instability



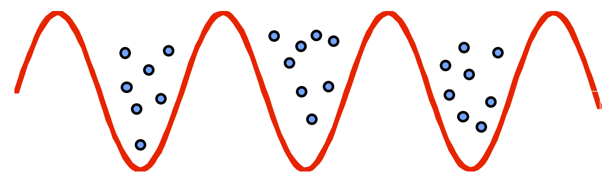
LDI Threshold ~ $k\lambda_D \quad \overline{\square_e \square_i}$

Wave-Particle ($k\lambda_D \sim v_e/v_\phi \sim 1$)

Electron Trapping



Wave frequency decreases ~ $\square^{1/2}$

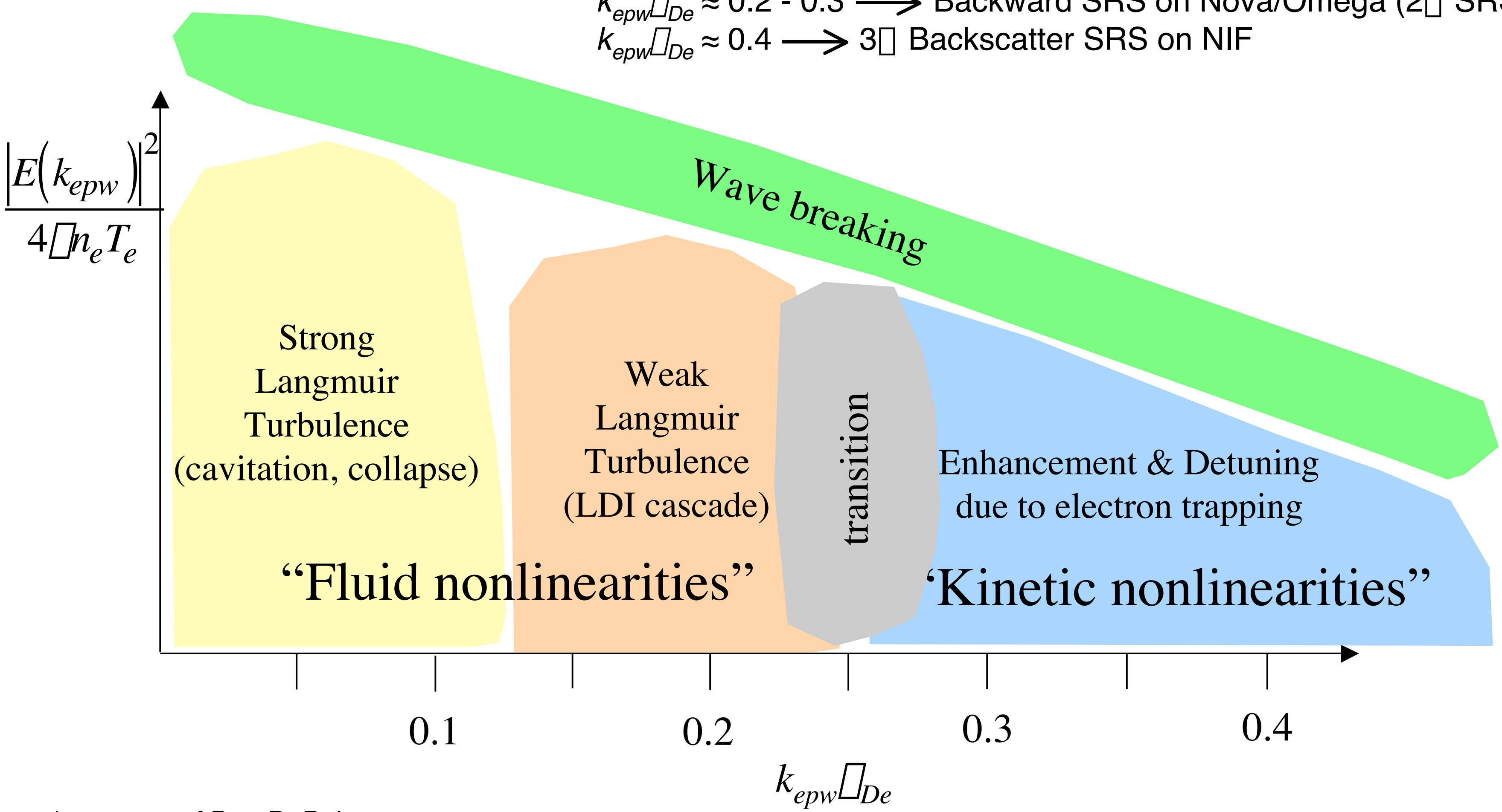


Loss of resonance due to detuning

Increasing $k\lambda_D$ →

A global theoretical understanding of Langmuir wave saturation physics is emerging*

$k_{epw}\lambda_{De} \approx 0.1 \longrightarrow$ Forward SRS on NIF
$k_{epw}\lambda_{De} \approx 0.2$ - $0.3 \longrightarrow$ Backward SRS on Nova/Omega (2ω SRS NIF?)
$k_{epw}\lambda_{De} \approx 0.4 \longrightarrow$ 3ω Backscatter SRS on NIF



* courtesy of Don DuBois

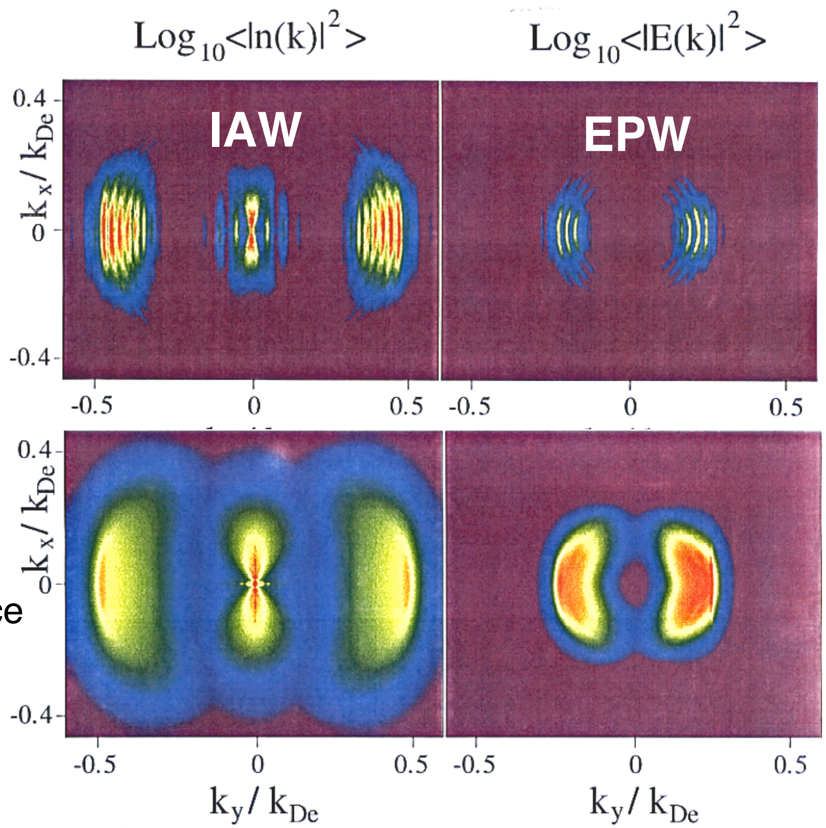
Similar picture might be made for ion wave physics

Determining the nature of Langmuir wave turbulence is important for understanding SRS saturation in some regimes

2-D SRS simulations

LDI cascade

Strong turbulence

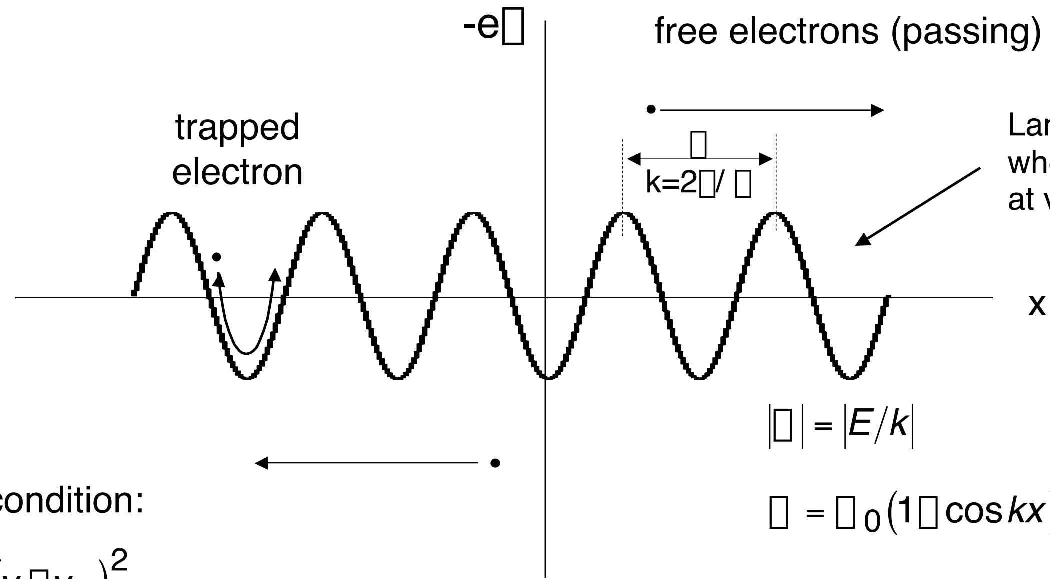


from Russell, Dubois, and Rose, *Phys. Plasmas* **6**, 1294 (1999).

- Needed to benchmark SRS models
- (□, k) spectra reveals the turbulence regime
- Experimentally challenging to measure (homogeneous system)
- Inhomogeneous (RPP) experiment *blurs* cascade spectrum† (□u/c_s > 0.05, □n/n > 0.01)
- Single hot spot experiment enables resolution of spectrum

† S. Depierreux *et al.,* PRL. **84**, 2869 (2000)
D.S. Montgomery, PRL **86**, 3686 (2001).

Overview on particle trapping effects



Langmuir wave is stationary when viewed in a frame traveling at v$_\square$ = □ / k (+x direction)

$$|\square| = |E/k|$$

$$\square = \square_0(1 \square \cos kx) = \square_0 \square \frac{1}{2} k^2 x^2 + \cdots \square$$

$$m_e \frac{d^2x}{dt^2} = \square e \frac{d\square}{dx} = \square ek\square_0 \sin kx \ \square \ \square ek(E/k)kx$$

$$\frac{d^2x}{dt^2} + \square_B^2 x = 0, \ \square_B = |ekE/m|$$

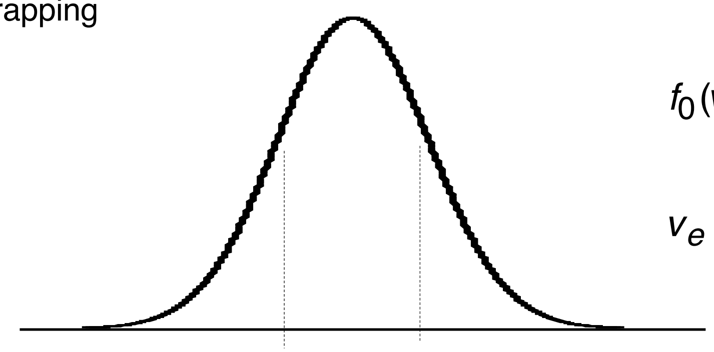
Trapping condition:

$$|e\square| > \frac{1}{2} m(v \square v_\square)^2$$

$v_\square = \square/k \equiv$ wave phase velocity

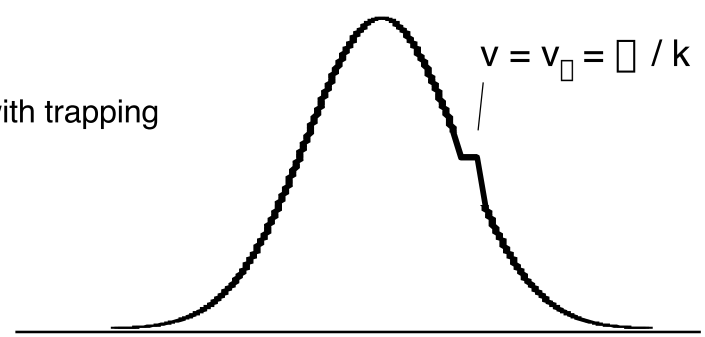
- linear Landau damping breaks down on a time scale $\square_B\square$>> 1 (no surfers, just skaters!)
- when $\square_B\square$>> 1, net Landau damping is zero! (nonlinear Landau damping)
- to conserve energy, $v_\square$ decreases as more particles are trapped (nonlinear frequency shift)

Particle trapping tends to flatten the distribution function near the wave phase velocity



$$f_0(v) = n\left(\frac{m_e}{2\pi KT}\right)^{1/2} \exp\left(\frac{-v^2}{2v_e^2}\right)$$

$v_e \equiv \sqrt{\dfrac{KT}{m_e}}$, definition of temperature in kinetic theory



$v_\phi / v_e >> 1$
few particles in distribution,
kinetic effects small

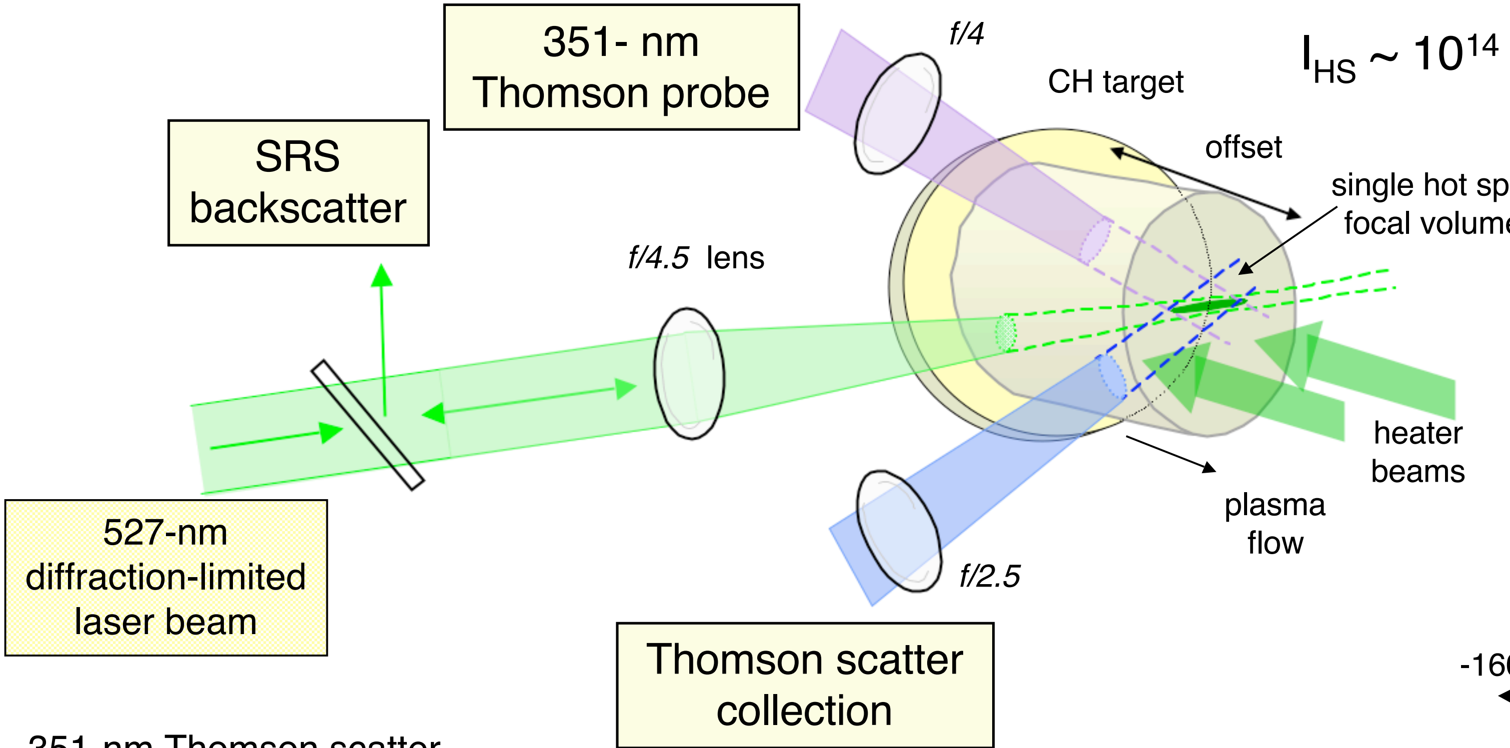
$v_\phi / v_e \sim 1$
wave interacts with particles in bulk distribution,
large kinetic effects [modify $f_0(v)$]

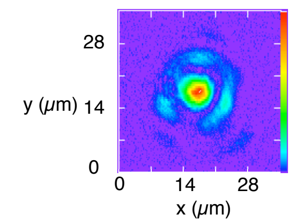
Landau damping rate:

$\gamma \sim \partial f_0 / \partial v$
reduced damping in vicinity of flattened $f_0(v)$

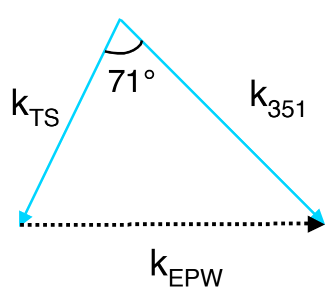
Single hot spot experiments are performed in well-characterized, simple plasma systems

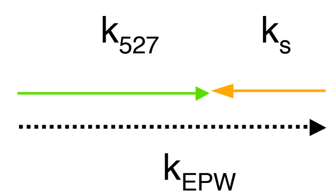
$I_{HS} \sim 10^{14}$ - 10^{16} W/cm^2

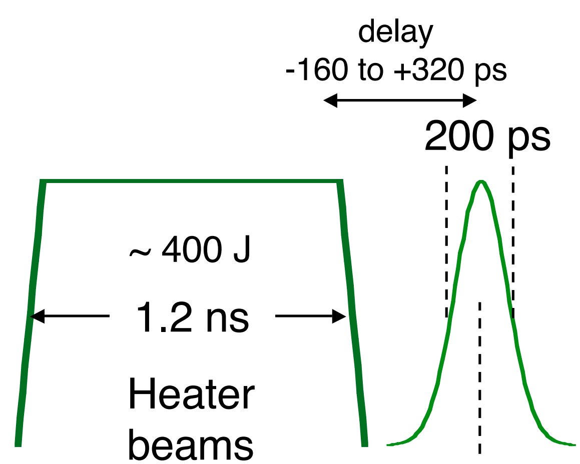




351-nm Thomson scatter from driven EPW

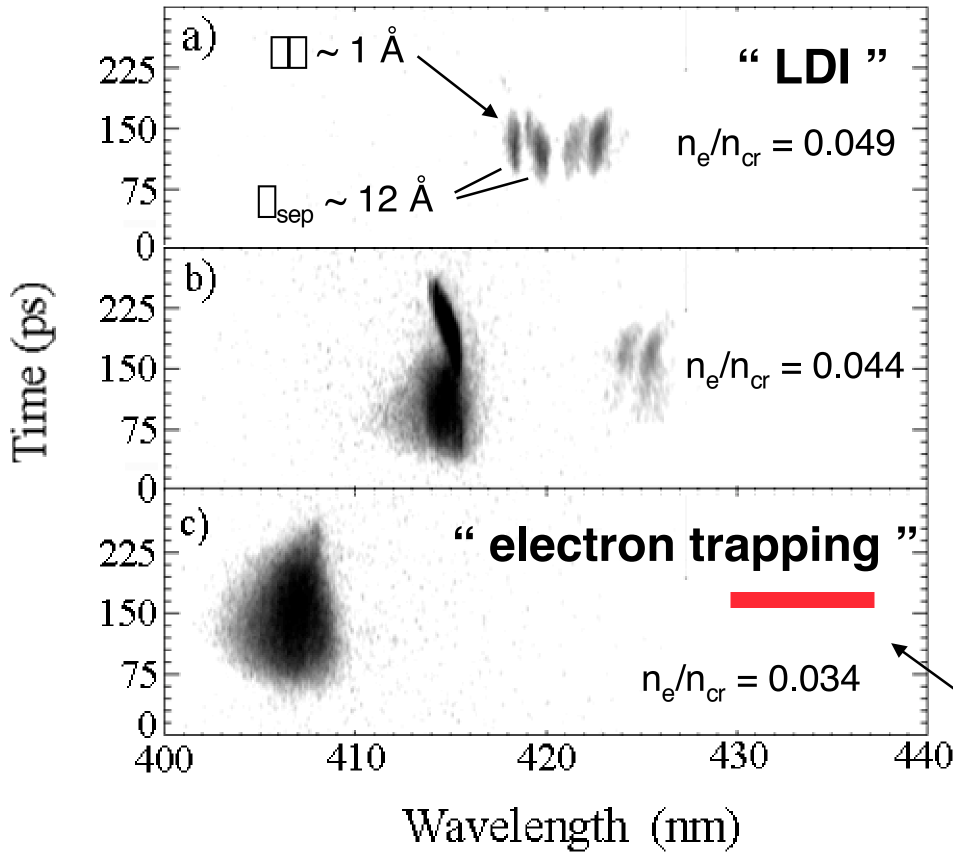




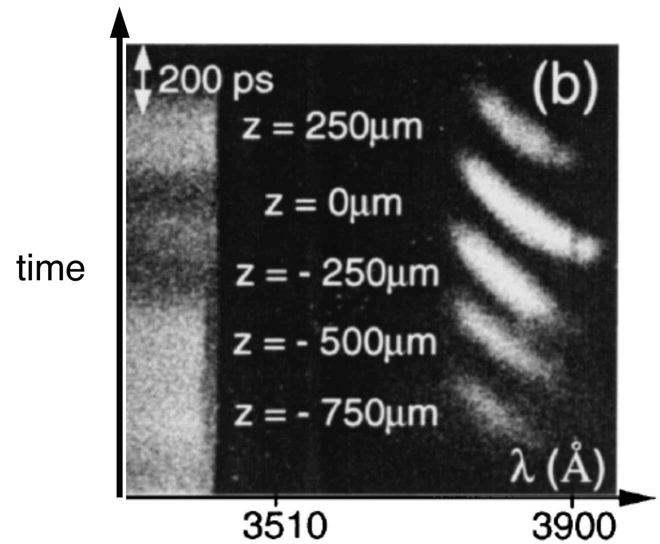


The Thomson scattered spectral features associated with LDI cascade and electron trapping are readily discerned in single hot spot experiments

Thomson EPW spectra from single hot spot experiment



Thomson EPW spectra from RPP experiment



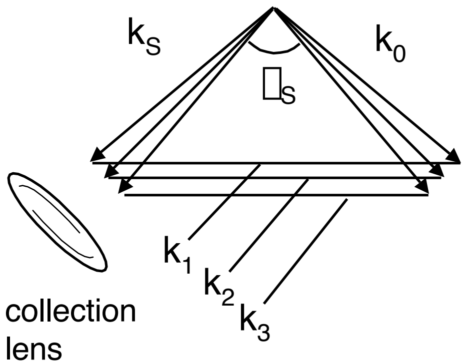
S. Depierreux *et al.*, *Phys. Rev. Lett.* **84**, 2869 (2000)

width of spectrum from RPP experiment

for trapping, ∂□/∂t >> □□/□t (occurs on sub-ps scale) so that broadening is observed rather than shifts

Single hot spot configuration allows measurement of discrete (□, k) steps associated with LDI cascade

wave-matching conditions



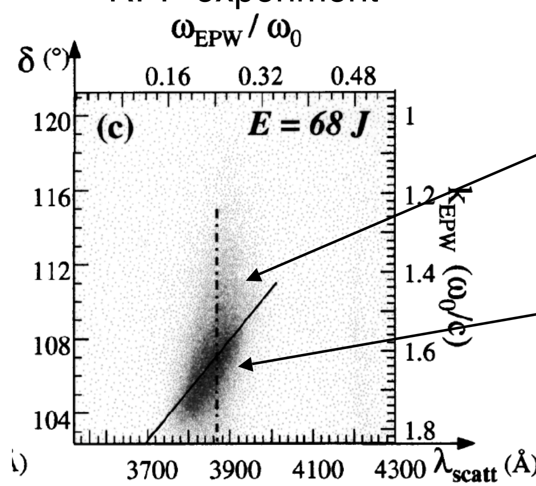
- frequency directly from $□_0 = □_s + □_{epw}$

- k_{epw} obtained from scattering angle
  - image collection lens plane
  - gated imaging spectroscopy
  - $□_s$, $□_s$ to obtain $□_{epw}$, k_{epw}

□k, □□ in excellent agreement with LDI theory

"□, k" spectra from RPP experiment

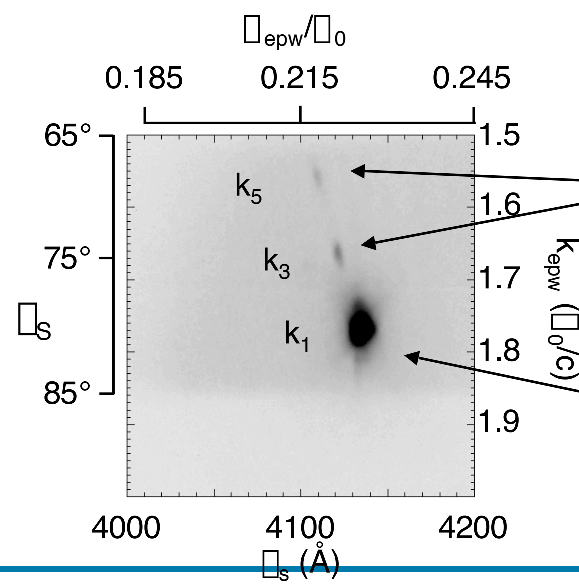
from S. Depierreux *et al.*, *Phys. Rev. Lett.* **89** (2002)



LDI cascade inferred

primary EPWs from SRS

SHS experiment



LDI cascade *fully resolved*

primary EPW from SRS

There are many other types of wave nonlinear phenomena that can occur

- Drift instabilities, fluid (MHD) instabilities…
- Wave steepening (harmonics)
- Wave breaking
- Shocks
- Solitons
- Self-focusing, wave collapse
- Wave turbulence, mode coupling
- Phase-space vortices (new nonlinear phenomena)
- All the possible interactions of waves in a magnetized plasma…
- ….
- ….

The whole zoo of processes is enough to keep scientists at several laboratories gainfully employed (job security)

Summary and Conclusions

- Applications of laser-plasmas
- Laser Fusion
- Review of oscillations and waves in plasmas
- Coupled Oscillators: parametric instabilities
- Some types of parametric instabilities important to laser fusion
- Particle trapping nonlinearities
- Examples from experiments
- Other types of nonlinear phenomena

Laser-plasma interaction physics is a challenging and exciting area of research