PLASMA PHYSICS AND COSMIC RAYS

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OUTLINE

1. General properties of cosmic rays

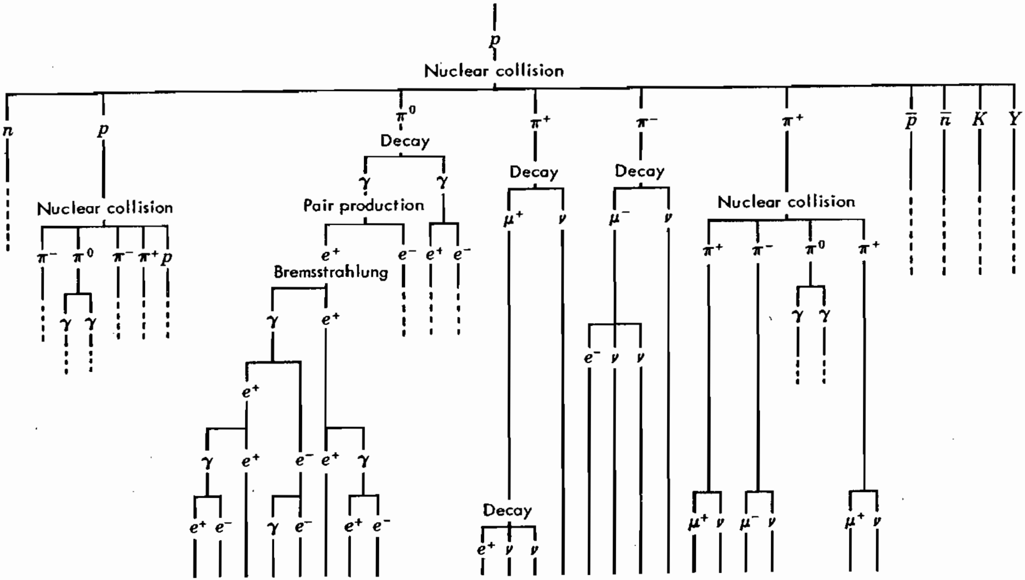
2. Pitch angle scattering of cosmic rays

3. Alfven wave instability

4. Model of sinks and sources in the ISM

5. Origin of cosmic rays

6. Shock Acceleration



p
Nuclear collision
n
p
π0
π+
π−
π+
p̄
n̄
K
Y
Nuclear collision
π− π0 π− π+ p
γ γ
Decay
γ γ
Pair production
e+ e− e+ e−
Bremsstrahlung
γ e+
e+
γ e+ e− e+ γ
e+ e− γ e− e+ e−
Decay
μ+ ν
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Nuclear collision
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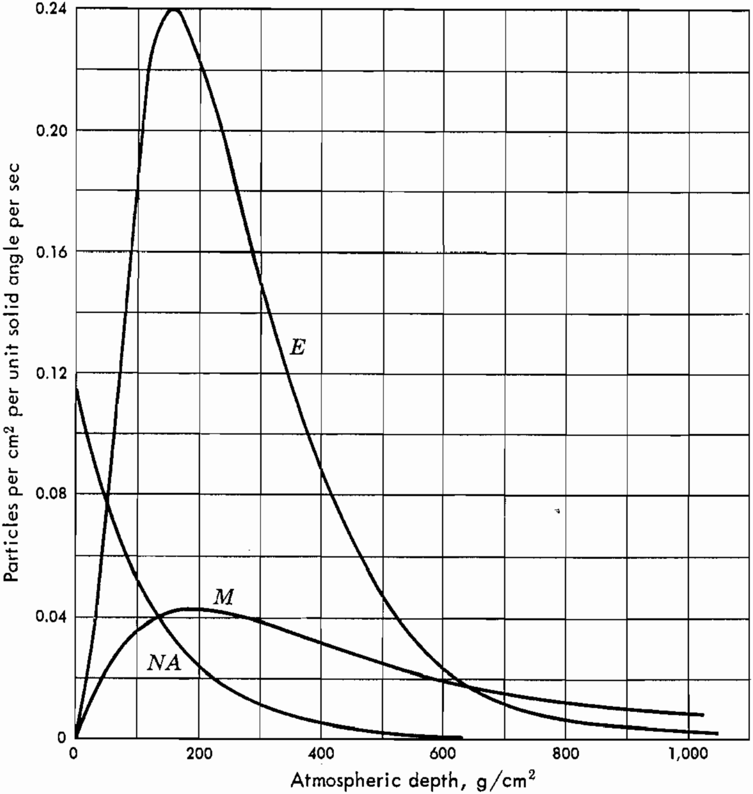
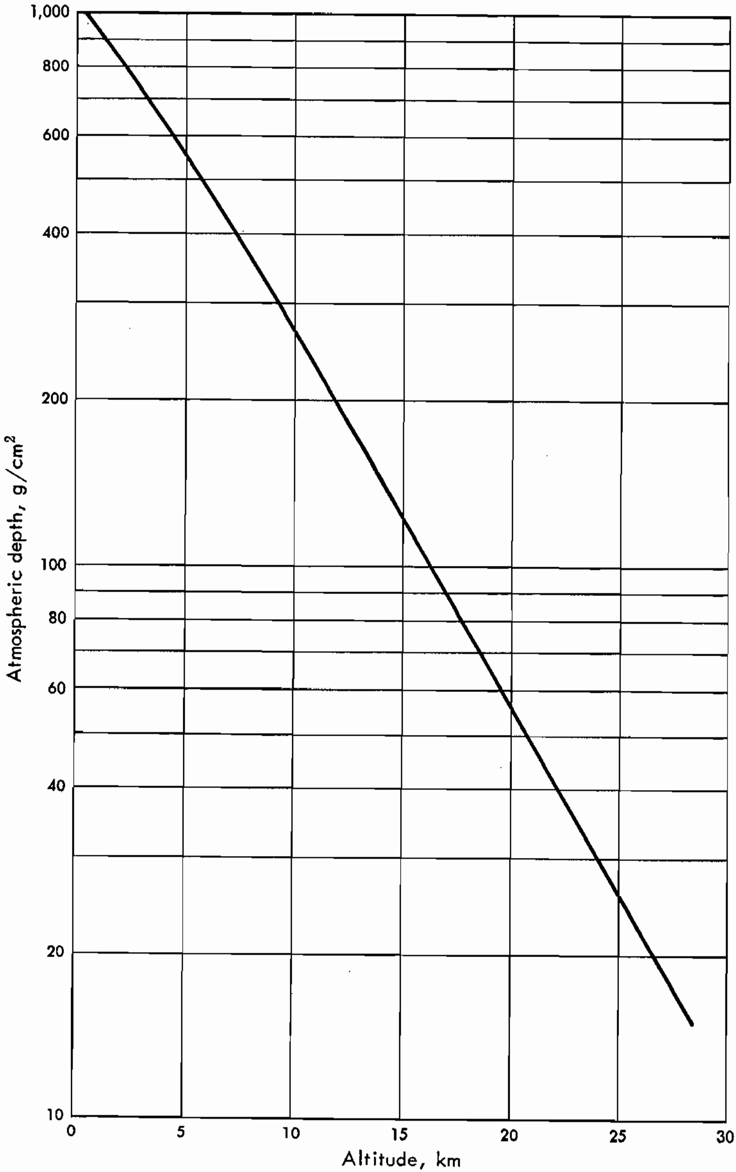
μ+ ν μ− ν μ+ ν


0.24
0.20
0.16
0.12
0.08
0.04
0
Particles per cm² per unit solid angle per sec
E
M
NA
0
200
400
600
800
1,000
Atmospheric depth, g/cm²


1,000
800
600
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40
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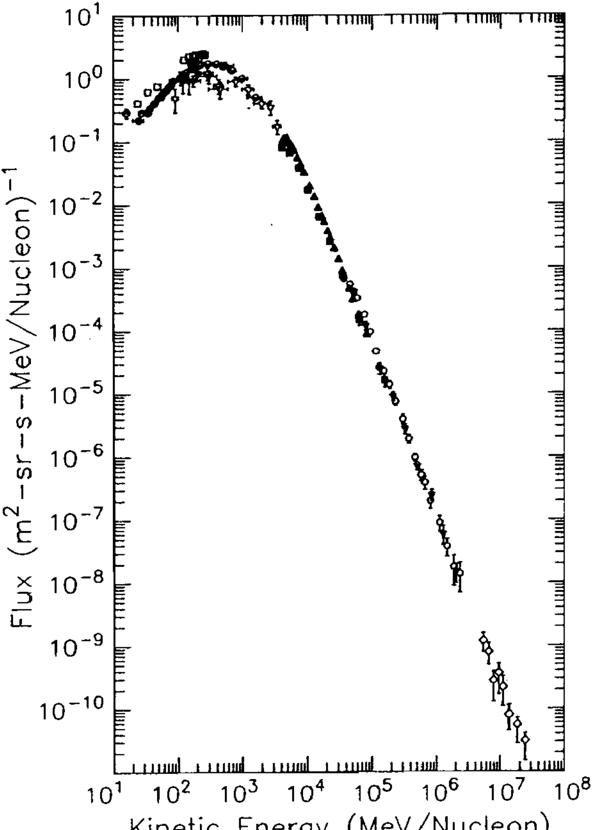
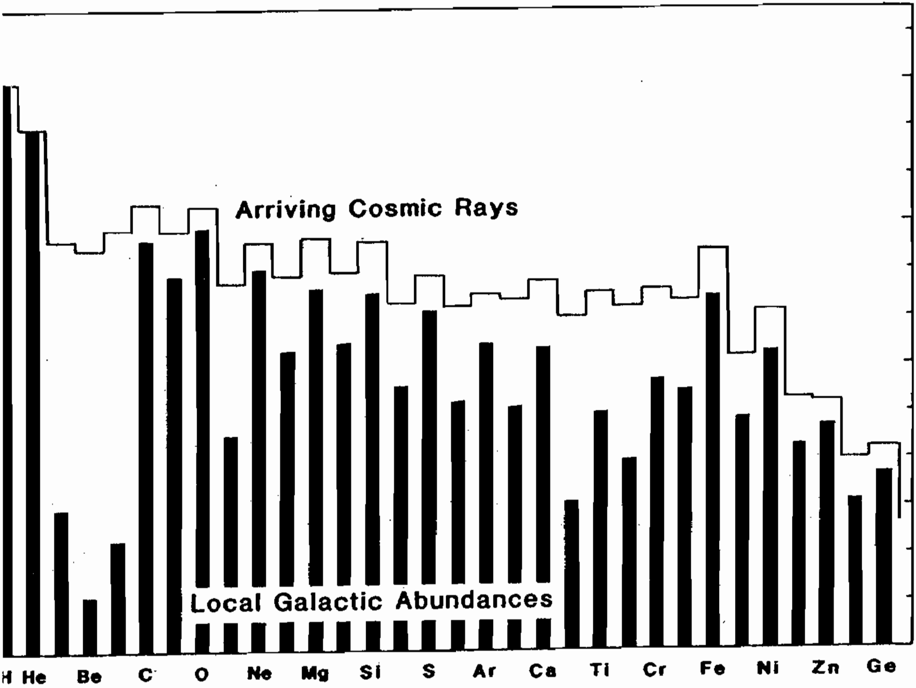
10
Atmospheric depth, g/cm²
0
5
10
15
20
25
30
Altitude, km



Flux $(m^2-sr-s-MeV/Nucleon)^{-1}$
Kinetic Energy (MeV/Nucleon)
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8


Arriving Cosmic Rays
Local Galactic Abundances
H He Be C O Ne Mg Si S Ar Ca Ti Cr Fe Ni Zn Ge

$$N(\epsilon) \sim \epsilon^{-2.7} \tag{1}$$

$$\Omega_{cr} = \frac{eB}{\gamma mc} = \frac{3 \times 10^{-2}}{\gamma} \mathrm{sec}^{\ -1} \tag{2}$$

$$\rho = \frac{c}{3 \times 10^{-2}} \epsilon(\mathrm{GeV}\ ) = 10^{12} \epsilon(GeV)\ \mathrm{cm} \tag{3}$$

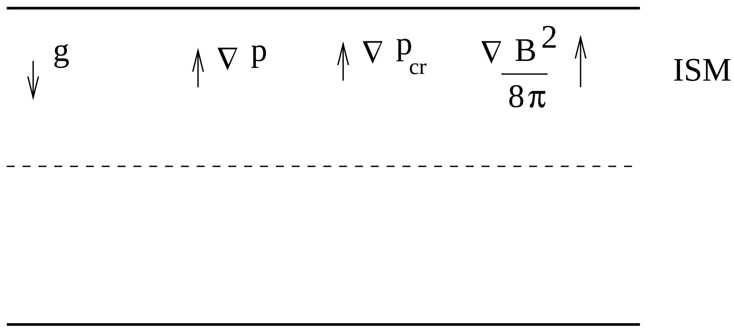


Figure 1: Cosmic ray support in the interstellar medium

$$j_{cr} = n_{cr} e v_D / c = \nabla p_{cr} / B \tag{4}$$

$$j_p = \frac{\nabla p}{B} \tag{5}$$

$$j_g = -\frac{\rho g}{B}$$

$$j_{tot} = j_{cr} + j_g + j_p = \frac{\nabla p_{cr} + \nabla p - \rho g}{B} \tag{6}$$

$$\frac{\nabla p_{cr} + \nabla p - \rho g}{B} = \frac{\nabla B}{4\pi} \tag{7}$$

$$\rho g = \nabla \left(p_{cr} + p + \frac{B^2}{8\pi} \right) \tag{8}$$

0.1 Scattering of cosmic rays by Alfven waves

$$\delta B_\perp = \hat{\mathbf{x}} \delta B \sin(kz - \omega t) \tag{9}$$

$$\mathbf{v} = \hat{\mathbf{y}} v_\perp \sin(\Omega t + \phi), \tag{10}$$

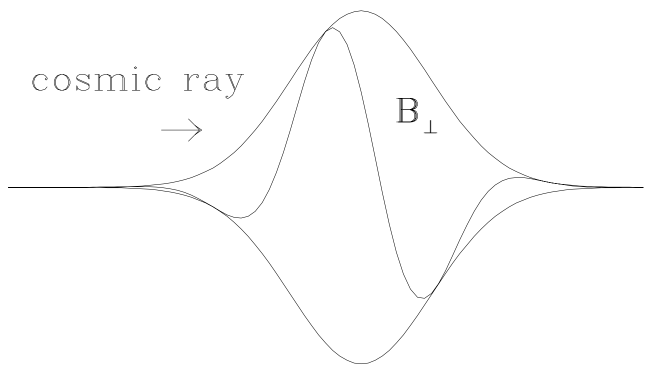


Figure 2: A cosmic ray and a wave packet

Let the cosmic ray z position be $v_z t$. Then

$$
\begin{aligned}
(\mathbf{v} \times \mathbf{B})_z &= -\hat{\mathbf{z}} v_\perp \delta B \\
&\times \sin(kz_0 + kv_z t) \sin(\Omega t + \phi)
\end{aligned}
$$

or

$$
\begin{aligned}
(\mathbf{v} \times \mathbf{B})_z &= \frac{1}{2} v_\perp \delta B \\
&\times \{\cos[(kv_z - \omega + \Omega)t + \phi] \\
&- \cos[(kv_z - \omega - \Omega)t - \phi]\}
\end{aligned}
$$

If $v_z > 0$ and

$$kv_z - \omega - \Omega \approx 0 \tag{11}$$

then the force does not average out, and the, the change in p_z due to this interaction is of order

$$\begin{aligned}
\Delta p_z &= e\int dt \left(\frac{\mathbf{v}\times\mathbf{B}}{c}\right)_z \\
&\approx \frac{1}{2}\frac{ev_\perp \delta B}{c} \times \frac{2\pi}{kv_z}\cos\phi' \\
&= \pi\frac{e\gamma v_\perp}{c}\frac{\delta Bmc}{eB}\cos\phi' \\
&= \pi p_\perp \sin\theta \left(\frac{\delta B}{B}\right)\cos\phi'
\end{aligned}$$

where we have let the wave packet have a length $L = 2\pi/k$ and where $\phi' = kz_0 - \phi$ is the relative phase between the cosmic ray and the wave.

$$\begin{aligned}
\delta(pc\cos\theta) &= = -p\sin\theta\delta\theta \\
&= \pi p\sin\theta\left(\frac{\delta B}{B}\right)\cos\phi'
\end{aligned}$$

or

$$\delta\theta = -\pi \frac{\delta B}{B} \cos\phi' \tag{12}$$

taking into account the random sign, the average of $(\delta\theta)^2$ per unit time is

$$\frac{(\Delta\theta)^2}{t} = \frac{\pi}{8}\Omega \left(\frac{\delta B}{B}\right)^2$$

since the cosmic ray encounters waves at the rate $c/\lambda \approx \Omega$.

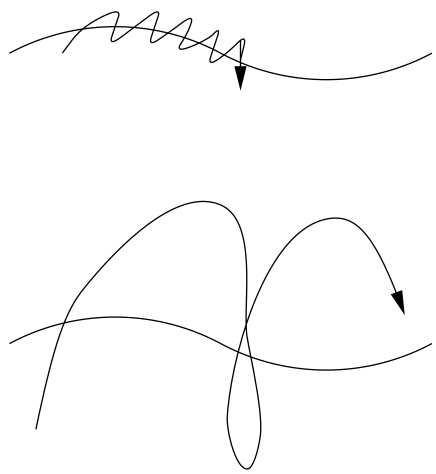


Figure 3: No change in pitch angle unless $\lambda \approx \rho$

Three remarks are worth making.

$$\delta\theta \approx \pm\frac{\delta B}{B} \tag{13}$$

$$kv \approx \Omega \tag{14}$$

or

$$\lambda/2\pi \approx \frac{v}{\Omega} \approx r_L \tag{15}$$

The second remark is: If we consider n waves in our wave packet we have

$$\frac{(\Delta\theta)^2}{t} = n\frac{\pi}{8}\Omega\left(\frac{\delta B}{B}\right)^2 \tag{16}$$

But for a smooth spectrum the amplitude is of the wave packet is obtained from a narrower k band $\Delta k \approx k/n$ so

$$\left(\frac{\delta B}{B}\right)^2 = \Delta k I(k) \tag{17}$$

and n drops out.

$$D_\theta = \frac{(\Delta\theta)^2}{2t} = \frac{\pi}{8}\Omega I \tag{18}$$

A third remark of considerable importance concerns the resonance condition

$$kv_z = kv\cos\theta = \Omega \tag{19}$$

or

$$\lambda/2\pi = r_L\cos\theta \tag{20}$$

and as θ goes through 90 degrees, λ goes to zero and we have a singular situation.

We will come back to this point.

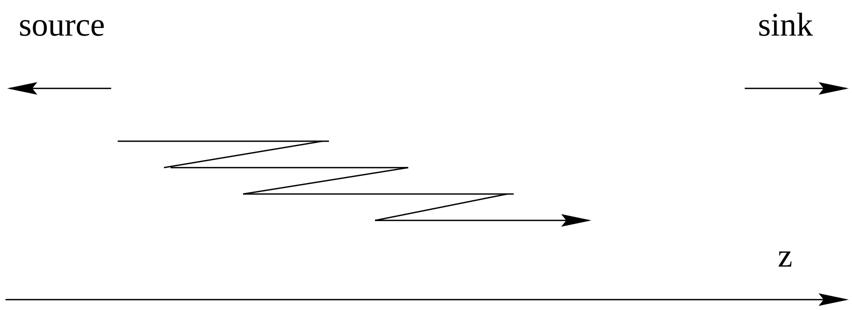


Figure 4: Why the mean speed of a cosmic ray is less than c

How do we connect the lifetime of the cosmic rays in the disc ($\approx 3 \times 10^6$ years). The anisotropy corresponds to a drift velocity v_D, then

$$\frac{v_D}{c} = \delta \approx 10^{-4} \tag{21}$$

and $v_D \approx$ ten kilometers per second.

If the mean free path for pitch angle scattering through

90 degrees is λ, then the lifetime of the cosmic rays is

$$\frac{L^2}{c\lambda} = 3 \times 10^6 \text{years} \tag{22}$$

which leads to $\lambda \approx 10$pc. or a ninety degree scattering time of 30 years.

From

$$\frac{v_{\parallel}}{\lambda} \approx \frac{(\Delta\theta)^2}{t} \approx \Omega \left(\frac{\delta B}{B}\right)^2 \tag{23}$$

we find that $(kI)^{1/2} \approx \delta B/B = 10^{-4}$.

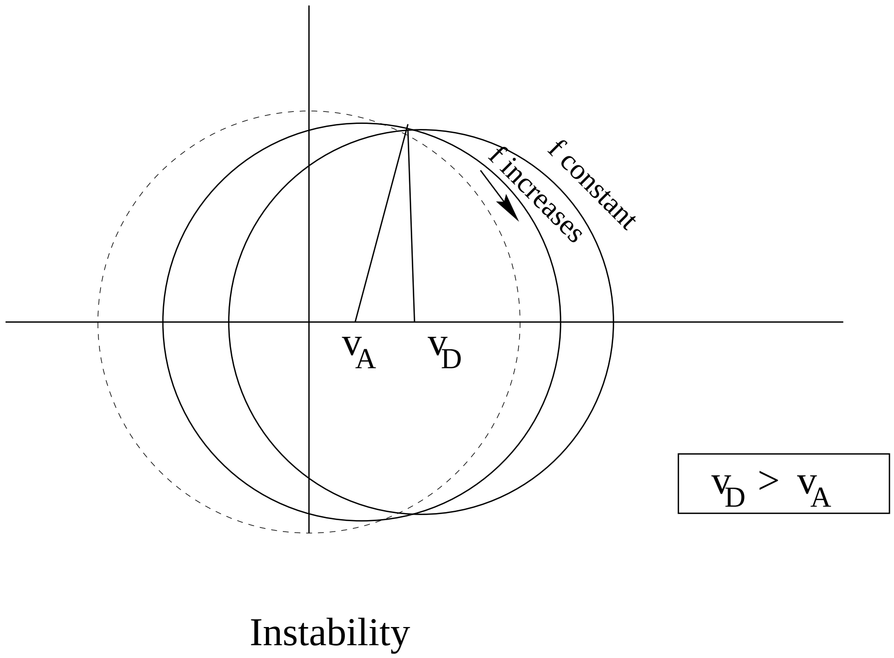


Figure 5: Position of the constant f surfaces relative to the pitch and scattering curves and instability

0.2 The Alfven wave instability

What is the origin of these Alfven waves that scatter cosmic rays?

Up to this point we have not discussed the source of the Alfven waves that scatter cosmic rays. There are a number of possible sources, interstellar turbulence, hot stars, moving magnetic stars and so forth. But surprisingly the main source seems to be the cosmic rays themselves. A detailed kinetic calculation involving the Vlasov equation for the cosmic rays shows that if the cosmic rays are sufficiently anisotropic they will render Alfven waves carried by the interstellar medium unstable. If the anisotropy is due to a bulk drift velocity v_D, and if $v_D > v_A$, then the waves are unstable.

Before we enter into the detailed calculation, of the expected growth rate for Alfven waves let us see if we can understand why cosmic rays should make Alfven waves unstable.

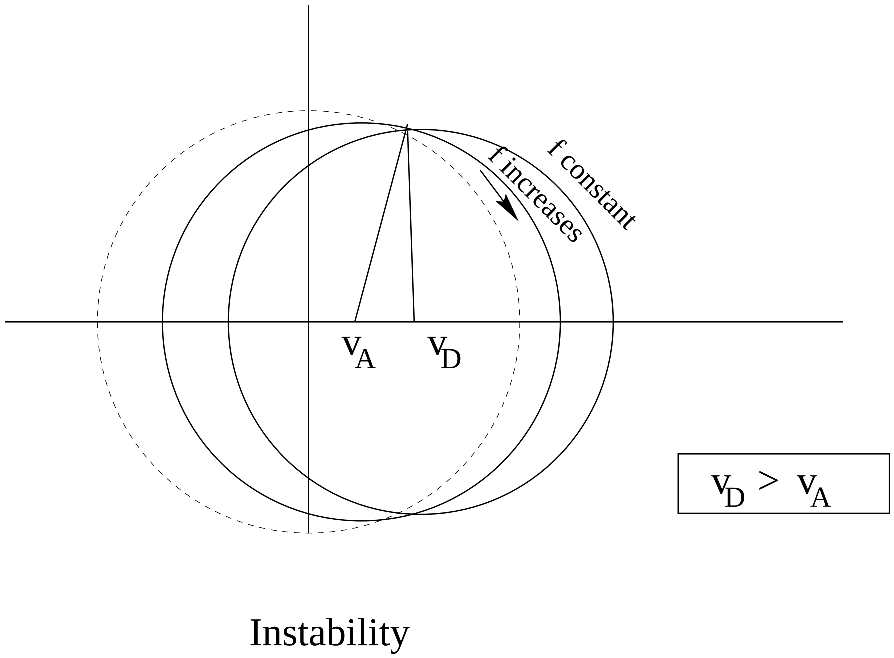


Figure 6: Position of the constant f surfaces relative to the pitch and scattering curves and instability

Suppose that the cosmic rays have a drift velocity of $V_D > V_A$, and that there are some small amplitude right moving Alfven waves present. As seen above, the waves lead to a pitch angle diffusion of the cosmic rays (in the wave frame) at the rate

$$D_\theta = \frac{(\Delta\theta)^2}{2t} \approx \Omega \left(\frac{\delta B}{B}\right)^2 \tag{24}$$

The time τ to isotropize these cosmic rays in this wave frame is

$$\tau = \frac{1}{D_\theta} = \frac{1}{\Omega(\delta B/B)^2}. \tag{25}$$

Before the scattering the cosmic rays would have a linear momentum

$$n_{cr} m v_D, \tag{26}$$

After the scattering, the momentum is

$$n_{cr} m v_A, \tag{27}$$

so the rate of the loss of cosmic ray momentum is

$$\frac{dP_{cr}}{dt} = \frac{n_{cr} m (v_D - v_A)}{\tau}. \tag{28}$$

But this corresponds to a rate of gain of the wave momentum,

$$2\gamma P_{wave} = 2\gamma \frac{(\delta B)^2}{v_A 8\pi} \tag{29}$$

since any wave momentum is equal to its wave energy divided by its phase velocity

Thus, equating these rates and taking $m = \Gamma_{cr} M$ where M is the rest mass of the cosmic ray, gives a growth rate

$$
\begin{aligned}
2\gamma &= \frac{n_{cr}\Gamma_{cr}(v_D - v_A)\Omega(\delta B/B)^2}{(\delta B)^2/8\pi v_A} \\
&= n_{cr}\frac{8\pi M}{B^2}(v_D - v_A)v_A\Omega_0
\end{aligned}
\tag{30}
$$

or

$$
\gamma = \frac{n_{cr}}{n}\left(\frac{v_D - v_A}{v_A}\right)\Omega_0 \tag{31}
$$

$$
\gamma = \Omega_0\frac{n_{cr}}{n} = 3 \times 10^{-2} \times 10^{-10} = \approx 10^{-4}\text{years}^{-1} \tag{32}
$$

The growth of the Alfven waves must be balanced by some damping rate Γ.

In a warm partially ionized medium.

$$\Gamma = \frac{1}{2}\nu_{in} \tag{33}$$

The drift velocity V_D for cosmic rays is thus set by

$$\gamma(v_D) = \Gamma \tag{34}$$

But in a fully ionized medium the only damping of Alfven waves is the process of nonlinear Landau damping of the waves,

$$\gamma_{NL} \approx \omega \left(\frac{\delta B}{B}\right)^2 \frac{v_i}{v_A} \tag{35}$$

To balance this we must have

$$\gamma(v_D) = \omega \left(\frac{\delta B}{B}\right)^2 \frac{v_i}{v_A} \tag{36}$$

This equation, combined with the pitch angle scattering requirement on $\delta B/B$ to balance the drift velocity against any flow from cosmic ray sources to sinks determines both v_D and $\delta B/B$.

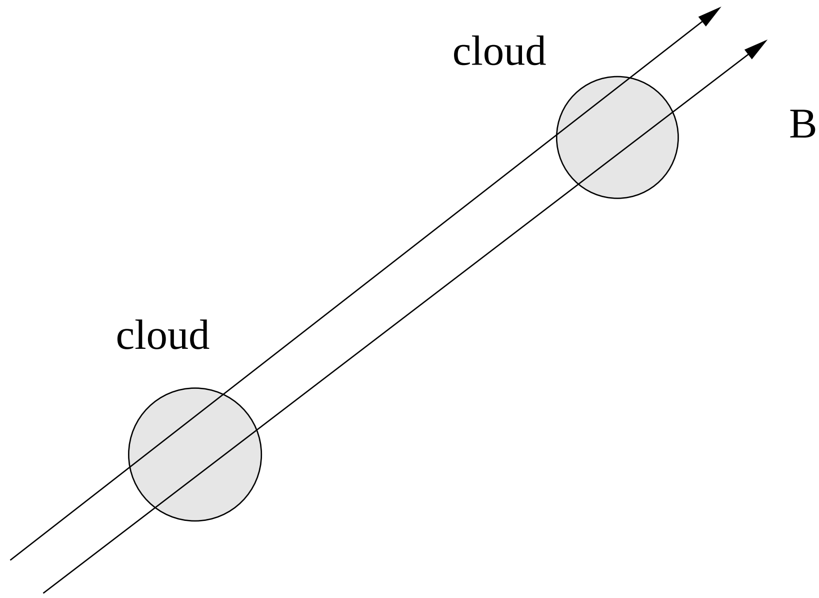


Figure 7: Cosmic rays passing through clouds

Cosmic ray spallation occurs in clouds but the propagation is set by the inter cloud medium.

The same result for the growth rate of the instability should, of course, be derived by an exact kinetic derivation.

I will briefly describe how such a derivation should go.

One starts with the relativistic Vlasov equation for f the cosmic ray distribution function in momentum space defined by.

$$\Delta N_{cr} = f(t, \mathbf{r}, \mathbf{p}) \tag{37}$$

The equation is

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + e\left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) \cdot \nabla_{\mathbf{p}} f = 0 \tag{38}$$

This important equation says that f

is constant along any cosmic trajectory no matter how complicated **E**,**B** and the cosmic ray orbit are.

We assume there is some zero order f_0 characterizing the equilibrium cosmic ray distribution with its anistropy and we imagine that an Alfven wave produces a perturbation f_1 in its distribution.

This perturbation is described by the Vlasov equation and we wish to calculate the perturbed cosmic ray current density

associated with the perturbation.

We substitute this in the dialectic tensor of the combined cosmic ray-background plasma defined by

$$\epsilon \cdot \mathbf{E} = \mathbf{E} + \frac{4\pi i}{\omega}(\mathbf{j_p} + \mathbf{j_{cr}}) \tag{39}$$

For an Alfven wave, $\mathbf{E}$ has the form

$$\mathbf{E} = Re\left(\hat{\mathbf{E}}e^{ikz-i\omega t}\right) \tag{40}$$

From the induction equation $(-i\omega/c)\mathbf{B_1} = -\mathbf{ik} \times \mathbf{E}$ or ,

$$\mathbf{B_1} = \frac{\mathbf{k}c \times \mathbf{E}}{\omega} \tag{41}$$

Therefore the linearization of Equation 38 is

$$\begin{aligned} -i\omega f_1 + ikv_z f_1 &+ \frac{e}{c}(\mathbf{v} \times \mathbf{B_0}) \cdot \nabla_{\mathbf{p}} f_1 \\ &= -e\left[\mathbf{E} + \frac{\mathbf{v} \times (\mathbf{k} \times \mathbf{E})}{\omega}\right] \cdot \nabla_{\mathbf{p}} f_0 \\ &= -e\left[(1 - \frac{kv_z}{\omega})\mathbf{E} + \frac{\mathbf{v} \cdot \mathbf{E}}{\omega}\mathbf{k}\right] \cdot \nabla_{\mathbf{p}} f_0 \end{aligned} \tag{42}$$

It is convenient to introduce cylindrical coordinates $p_\perp, \phi, p_z$, in $\mathbf{p}$ space. f_0 depends only on $p_\perp$ and p_z and is independent of ϕ, but f_1 depends on ϕ as well as $\mathbf{x}$ and $\mathbf{p}$. In these coordinates we have

$$\frac{e}{c}\mathbf{v} \times \mathbf{B_0} \cdot \nabla_{\mathbf{p}} f_1 = -\frac{ev_\perp B_0}{cp_\perp}\frac{\partial f_1}{\partial \phi} = -\Omega\frac{\partial f_1}{\partial \phi} \tag{43}$$

where $\Omega = v_\perp B_0/cp_\perp = eB_0/\gamma_R mc$ is the relativistic cyclotron frequency.

On the right-hand side of this equation $\mathbf{E} \cdot \nabla_p f_0 = E\cos\phi \partial f_0/\partial p_\perp$ and $\mathbf{E} \cdot \mathbf{v} = Ev_\perp \cos\phi$.

With these simplifications, The equation reduces to

$$\begin{aligned}
[-i\omega + ikv_z]f_1 &- \Omega\frac{\partial f_1}{\partial\phi} \\
&= -e\left[\left(1 - \frac{kv_z}{\omega}\right)\frac{\partial f_0}{\partial p_\perp} + \frac{kv_\perp}{\omega}\frac{\partial f_0}{\partial p_z}\right] E\cos\phi \\
&= -eAE\cos\phi \qquad (44)
\end{aligned}$$

where

$$A = \left(1 - \frac{kv_z}{\omega}\right)\frac{\partial f_0}{\partial p_\perp} + \frac{kv_\perp}{\omega}\frac{\partial f_0}{\partial p_z} \qquad (45)$$

A is an abbreviation for the bracketed expression

This equation is a simple differential equation in ϕ for f_1

. Its solution is

$$\begin{aligned}
f_1 &= -eE\frac{1}{2}\frac{iAe^{i\phi}}{\omega - kv_z + \Omega} - eE\frac{1}{2}\frac{iAe^{-i\phi}}{\omega - kv_z - \Omega} \\
&= f_+ + f_- \qquad (46)
\end{aligned}$$

The perturbed resonant current from the right moving cosmic rays is

$$\mathbf{j}_{1r} = \frac{-ie^2}{4} \int \left(\frac{v_\perp A}{\omega - kv_z + \Omega} \right) d^3\mathbf{p}(\hat{\mathbf{x}} + i\hat{\mathbf{y}})E \tag{47}$$

The electromagnetic equations reduce to

$$\frac{k^2c^2}{\omega^2} = \epsilon_0 + \epsilon_{xx}^{cr} \tag{48}$$

and

$$\epsilon_{xx}^{cr} = i\frac{4\pi j_{1x}}{\omega E} = 2\frac{e^2}{4}\frac{4\pi}{\omega} \int \frac{Av_\perp}{\omega - kv_z + \Omega} d^3\mathbf{p} \tag{49}$$

To lowest order

$$\omega_0 = kv_A \tag{50}$$

To next order the cosmic ray contribution to ϵ then produces a change in ω away from ω_0. That is, $\omega = \omega_0 + \omega_1$. Substituting this the above equation we get

$$-\frac{2\omega_1 k^2 c^2}{\omega_0^3} = -\frac{2\omega_1}{\omega_0}\frac{c^2}{v_A^2} = \epsilon_{xx}^{cr} \tag{51}$$

$\omega_1 = i\gamma$ is imaginary so combining these results we get

$$\gamma = \pi^2 e^2 \frac{v_A^2}{c^2}\int v_\perp \left[\left(1 - \frac{kv_z}{\omega}\right)\frac{\partial f_0}{\partial p_\perp} + \frac{kv_\perp}{\omega}\frac{\partial f_0}{\partial p_z}\right]\delta(kv_z - \Omega)d^3\mathbf{p} \tag{52}$$

We can exactly evaluate γ given by for a cosmic ray distribution which has a bulk velocity v_D and is isotropic in a frame moving with this velocity with a power law

$$F = \frac{a}{p^r} \tag{53}$$

($r = 4.7$ for a -2.7 power law energy spectrum.)

The answer is

$$\begin{aligned}\Gamma &= \frac{\pi^2 e^2}{kp_1}\frac{v_A^2}{c^2}N_{cr}(p > p_1)C_r\frac{v_D - v_A}{v_A}\\ &= \frac{\pi}{4}\Omega_0\frac{N_{cr}(p > p_1)}{n}C_r\frac{v_D - v_A}{v_A}\end{aligned} \tag{54}$$

where

$$C_r = \frac{r-3}{r-2} \tag{55}$$

(See Kulsrud and Cesarsky 1971.) For a spectral index of 2.7,

$r = 4.7$ and $C_{4.7} = 1.7/2.7 = 0.6$ so $(\pi/4)C_{4.7} = 0.5$.

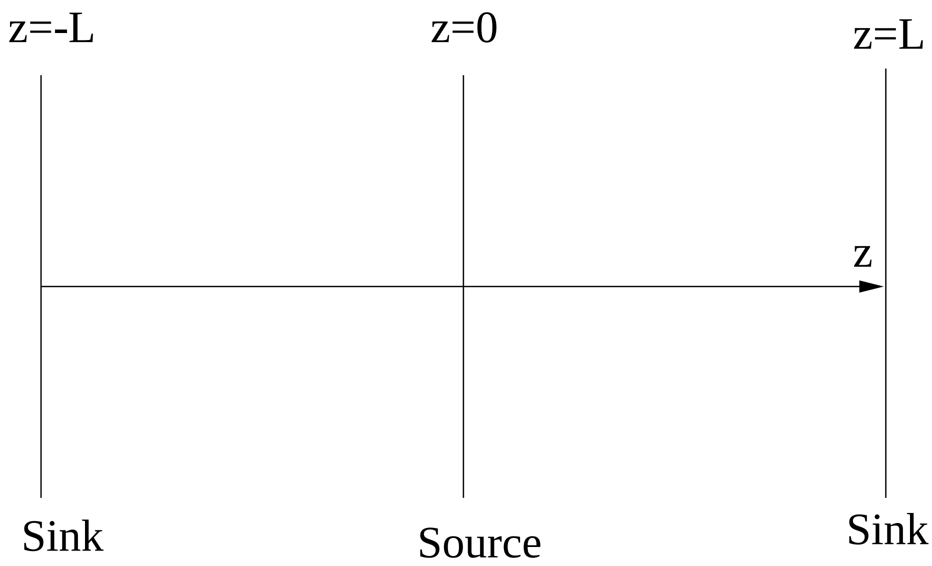


Figure 8: The source and sinks for the model

0.3 A Model for Cosmic Ray Propagation with Sources and Sinks

$$\left(\frac{\delta B}{B}\right)^2 = kI = \mathcal{E}(k) \tag{56}$$

Let the space dependence of $f(\mu, z)$ at $z = L/2$

be appoximated by

$$v_z \frac{\partial f}{\partial z} = -\frac{\mu v f(\mu)}{L} \tag{57}$$

From quasi-linear theory the pitch angle scattering equation, with $f(\mu)$ is $f(\mu, z)$ can be written

$$\frac{\partial f}{\partial t} - \frac{\mu v f}{L} = \Omega \frac{\pi}{4} \frac{\partial}{\partial \mu} \left[\mathcal{E}(\mu)(1 - \mu^2) \frac{\partial f}{\partial \mu} \right] \tag{58}$$

$\mathcal{E}(\mu) = \delta B/B$ is the relative magnetic fluctuation energy for waves at $k(\mu) = \Omega/v\mu$ in resonance with the cosmic rays

$$\mathcal{E}(\mu) = \mathcal{E}[|k(\mu)|] = \mathcal{E}(\Omega/v|\mu|) \tag{59}$$

Let

$$f(\mu, p) \approx N_{cr}(p > p_1)F(\mu) \tag{60}$$

Then the growth rate of waves resonant with μ

$$\begin{aligned} \gamma(\mu) &= \gamma(k) \\ &= \pi^2 e^2 \frac{v_A^2}{c^2} \int \frac{1-\mu^2}{p v_A} \frac{\partial f}{\partial \mu} v^2 \frac{\delta(\mu - \mu_c)}{kv} d^3p \end{aligned} \tag{61}$$

where $\mu_c = \Omega / kv$

This must balance the nonlinear damping γ_{NL} so

$$\gamma(\mu) = 0.3 \frac{\Omega}{\mu} \frac{v_i}{c} \mathcal{E} \tag{62}$$

The problem has been reduced to finding $F(\mu)$ the cosmic ray angular distribution function, and $\mathcal{E}(\mu)$ the relative energy of the Alfven waves resonant with the cosmic rays at the pitch angle μ,

versus μ.

$$\mathcal{E} = 1.29\sqrt{\frac{c^2}{v_i v_A}\frac{r_L}{L}\frac{N_{cr}}{n^*}}\mu\sqrt{(1-\mu^2)} \tag{63}$$

$$\frac{\partial F}{\partial \mu} = 0.49\sqrt{\frac{v_i v_A}{c^2}\frac{n^*}{N_{cr}}\frac{r_L}{L}}\frac{1}{\mu\sqrt{(1-\mu^2)}} \tag{64}$$

The problem at small μ is resolved by mirror reflection by Alfven waves if $\mu < mu_c$

$$\mu_c < \frac{\delta B}{B} = \sqrt{\mathcal{E}_1} \tag{65}$$

A detailed analysis by G. Felice and myself leads to the following result.

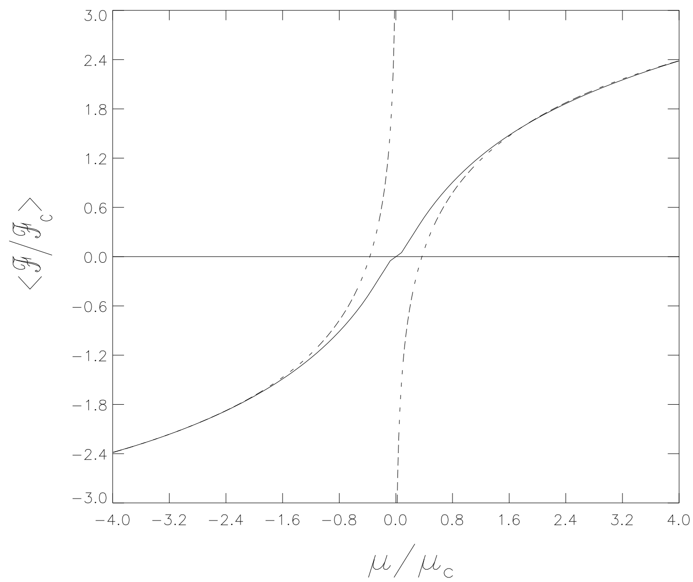


Figure 9: The cosmic ray distribution function near $\theta = 90$ $\mathcal{F}$ stands for the $F(\mu) - F(0)$ normalized to its variation in the boundary layer $\mu < \mu_c$. the dotted line represents the analytic solution without mirroring, and the solid line is the true distribution including mirroring.

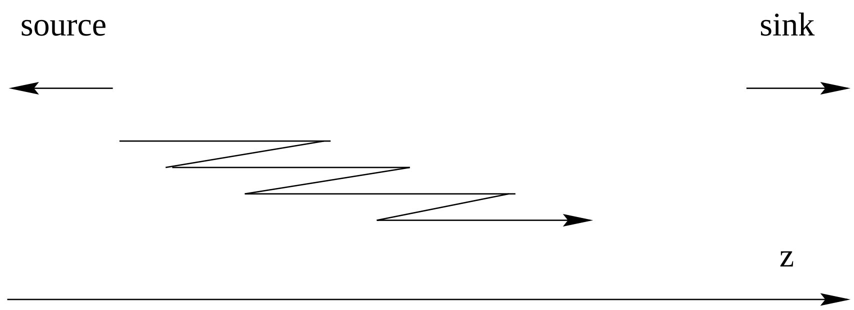


Figure 10: Why the mean speed of a cosmic ray is less than c

0.4 Cosmic Ray Pressure and Energy

Since cosmic rays have a pressure gradient along the magnetic field a drift velocity results. This drift velocity produces waves and the momentum of these waves is transferred to the interstellar medium. As a result the cosmic rays lose momentum at a rate give by

$$\frac{\partial P_{cr}}{\partial t} + V \cdot \nabla P_{cr} = -\frac{4}{3}(\nabla \cdot V) P_{cr} \tag{66}$$

Correspondingly the cosmic ray distribution function changes

and the effective equation for the cosmic ray distribution

is

$$\begin{aligned}\frac{\partial f}{\partial t} &+ v \cdot \nabla f \\ &= -\nabla \cdot (D\mathbf{nn} \cdot \nabla f) + \frac{1}{3}(\nabla \cdot \mathbf{V}) p \frac{\partial f}{\partial p}\end{aligned}$$

This equation is important for shock acceleration.

0.5 Fermi Acceleration and Shock Acceleration of Cosmic Rays

In 1949 Fermi (Fermi 1949) developed an origin theory for cosmic rays that involved a new theory of acceleration of high energy particles. He envisioned that cosmic rays in interstellar space would collide with moving clouds and in the collision the energy of the clouds would be gradually transferred to the cosmic rays.

$$\delta\epsilon = \frac{2vu\epsilon}{c^2} \approx \frac{2u}{c} \tag{67}$$

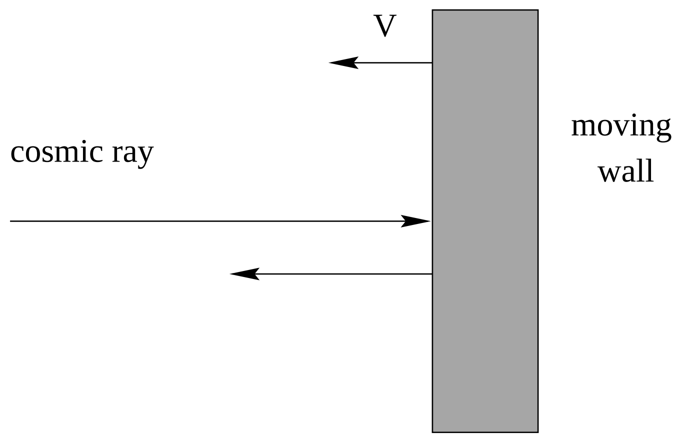


Figure 11: A cosmic ray reflected from a moving wall

$$\Delta \ln \epsilon = \Sigma \frac{u_i}{c} \tag{68}$$

$$\Delta \ln \epsilon = \Sigma \frac{u_i^2}{c^2} = NB \tag{69}$$

where B is the average change per collision.

If the mean time between encounters is τ, then the energy after a time t would be

$$\epsilon = \epsilon_1 e^{Bt/\tau} \tag{70}$$

where ϵ_1 is the initial energy.

Now, Fermi was aware that cosmic rays probably have a finite lifetime T in the interstellar medium.

$$dn = -n_0 e^{-t/T} \frac{dt}{T} \tag{71}$$

This leads to a distribution of cosmic ray energies given by

$$n_o e^{-t/T} \frac{dt}{T} = n_0 \left(\frac{\epsilon_1}{\epsilon} \right)^{\tau/BT} \frac{dt}{T d\epsilon} d\epsilon \tag{72}$$

which yields

$$dn \sim \frac{\epsilon_1^{\tau/BT}}{\epsilon^{1+\tau/Bt}} d\epsilon \tag{73}$$

a power law distribution ϵ^{-r} with exponent r.

$$r = 1 + \frac{\tau}{BT} \tag{74}$$

THE ORIGIN OF COSMIC RAYS

The most likely source is supernovae since a large amount of energy (10^{41} ergs/sec) is needed to replace them in 3 million years. Indeed very energetic electrons are detected by their synchrotron radiation in expanding supernova remnants.

But there is a problem of adiabatic deceleration in an expanding supernova remnant.

Cosmic rays cannot escape along the open field lines because of the Alfven wave instability.

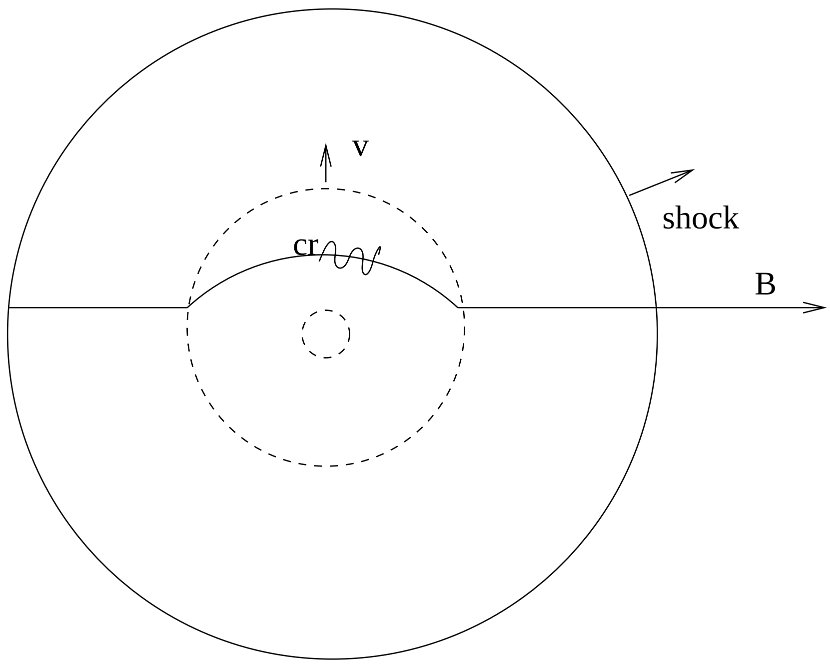


Figure 12: A cosmic ray trapped in expnding supernova remnant

Thus, if cosmic rays are produced initially at the supernova surface,

their individual energies will decrease

by $R_{max}/R_{min} \approx 10^6$

and correspondingly more total energy

must be produced by the same factor.

This is avoided if the cosmic ray are produced later at the supernova shock when the remnant is big and adiabatic deceleration is small.

This was first recognized about 1980, and seemed to solve the origin problem.

SHOCK ACCERATION

Shock acceleration occurs by first order Fermi acceleration where the cosmic ray always collides with oncoming mirrors which actually are Alfven waves.

Also, the lifetime is coorelated with the rate of acceleration so that the Fermi exponent $r = 1 + \tau/BT$ is of order unity, actually it is two.

To see this consider what happens when cosmic rays cross a shock wave.

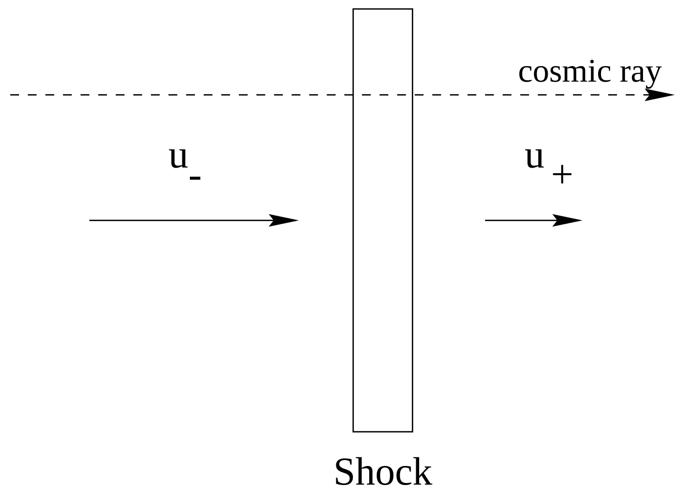


Figure 13: A cosmic ray crossing a strong shock

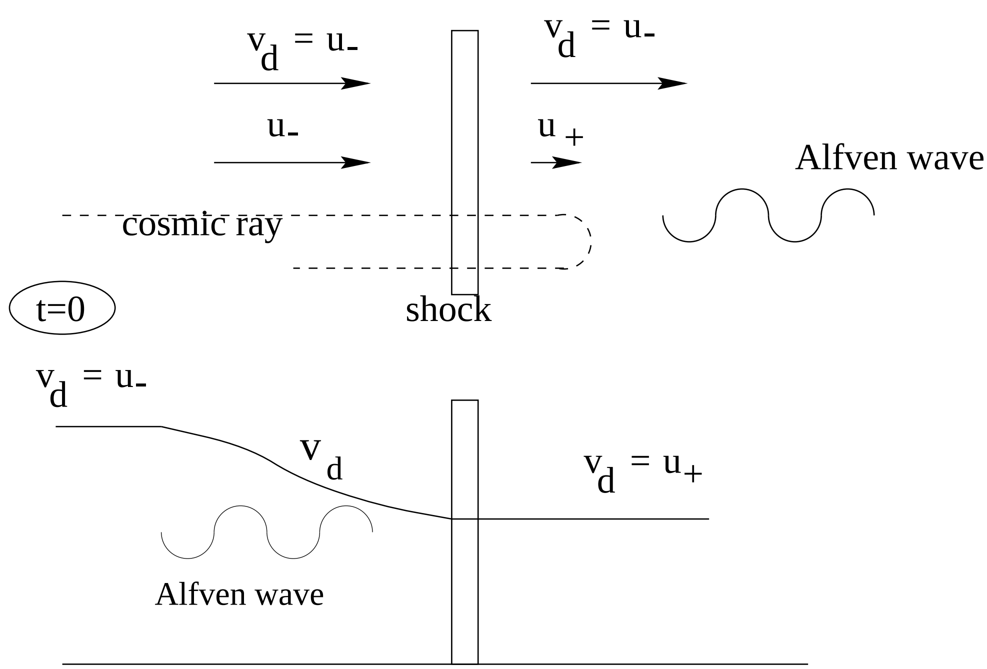


Figure 14: Excitation of Alfven waves and the modification of the cosmic ray distribution function

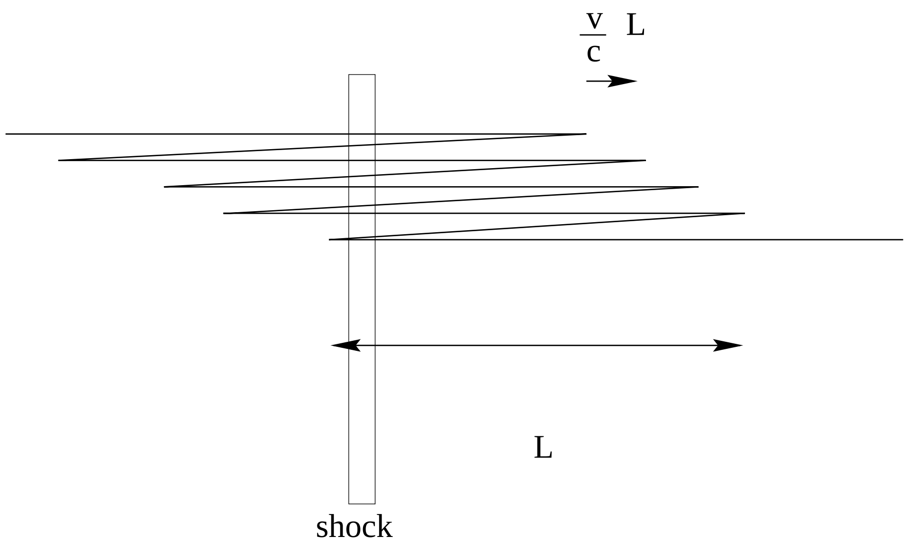


Figure 15: The cosmic ray cross the shock $\approx c/v_A$ times

This picture can be verified analytically in terms of our equations.

The cosmic ray kinetic equation from before was

$$\frac{\partial f}{\partial t} + v \cdot \nabla f = -\nabla \cdot (D\mathbf{nn} \cdot \nabla f) + \frac{1}{3}(\nabla \cdot \mathbf{V})p\frac{\partial f}{\partial p}$$

For a shock this becomes

$$v\frac{\partial f}{\partial x} - \frac{\partial}{\partial x}\left(D\frac{\partial f}{\partial x}\right) = \frac{1}{3}(u_+ - u_-)\delta(x)p\frac{\partial f}{\partial p} \qquad (75)$$

Integrating it across the shock at $x = 0$ we find the jump in the gradient of f

$$-D\left(\frac{\partial f_+}{\partial x} - \frac{\partial f_-}{\partial x}\right) = \frac{1}{3}(u_+ - u_-)p\frac{\partial f}{\partial p} \qquad (76)$$

where $-$ is upstream and $+$ is downstream.

First upstream.

$$u_- \frac{\partial f}{\partial x} - \frac{\partial}{\partial x}\left(D \frac{\partial f}{\partial x}\right) = 0 \tag{77}$$

The solution is

$$f = f_- + (f_0 - f_-)e^{xu_-/D} : \quad x < 0 \tag{78}$$

where $f_0 = f(p, 0)$. and We assume $f(x, P) \rightarrow f_-(p)$ and $\partial f / \partial x \rightarrow 0$ as $x \rightarrow -\infty$.

Now downstream.

$$u_+ \frac{\partial f}{\partial x} - \frac{\partial}{\partial x}\left(D \frac{\partial f}{\partial x}\right) = 0 \tag{79}$$

The solution is

$$f(p, x) = f_0(p) : \quad x > 0 \tag{80}$$

a constant since the $e^{xu_+/D}$ homogeneous part of the solution blows up as $x \rightarrow \infty$, Substituting these into

$$-D\left(\frac{\partial f_+}{\partial x} - \frac{\partial f_-}{\partial x}\right) = \frac{1}{3}(u_+ - u_-)p\frac{\partial f}{\partial p} \qquad (81)$$

we have

$$(f_0 - f_-)u_- = \frac{(u_+ - u_-)}{3}p\frac{\partial f_0}{\partial p} \qquad (82)$$

Let s be is the compression ratio of the shock, (i.e., $n_+ = sn_-$). It is of order 4 for a strong shock.

Then

$$p\frac{\partial f_0}{\partial p} + qf_0 = qf_- \qquad (83)$$

where $q = 3s/(s-1)$. The solution of this equation is

$$f_+ = f_0 = \frac{q}{p^q}\int_0^p f_- p'^{(q-1)} dp' \qquad (84)$$

This equation gives $f_0(p)$ the cosmic ray distribution function downstream in terms of the upstream function

$f_(p)$

If f_0 is very steep function of p than the downstream function is a pure power law with exponent q. For a strong shock has with q=4, corresponding to an energy spectrum with a minus two index.