Turbulent Self-Organization and Magnetic Relaxation in Plasmas

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P-24 Plasma Physics Summer School

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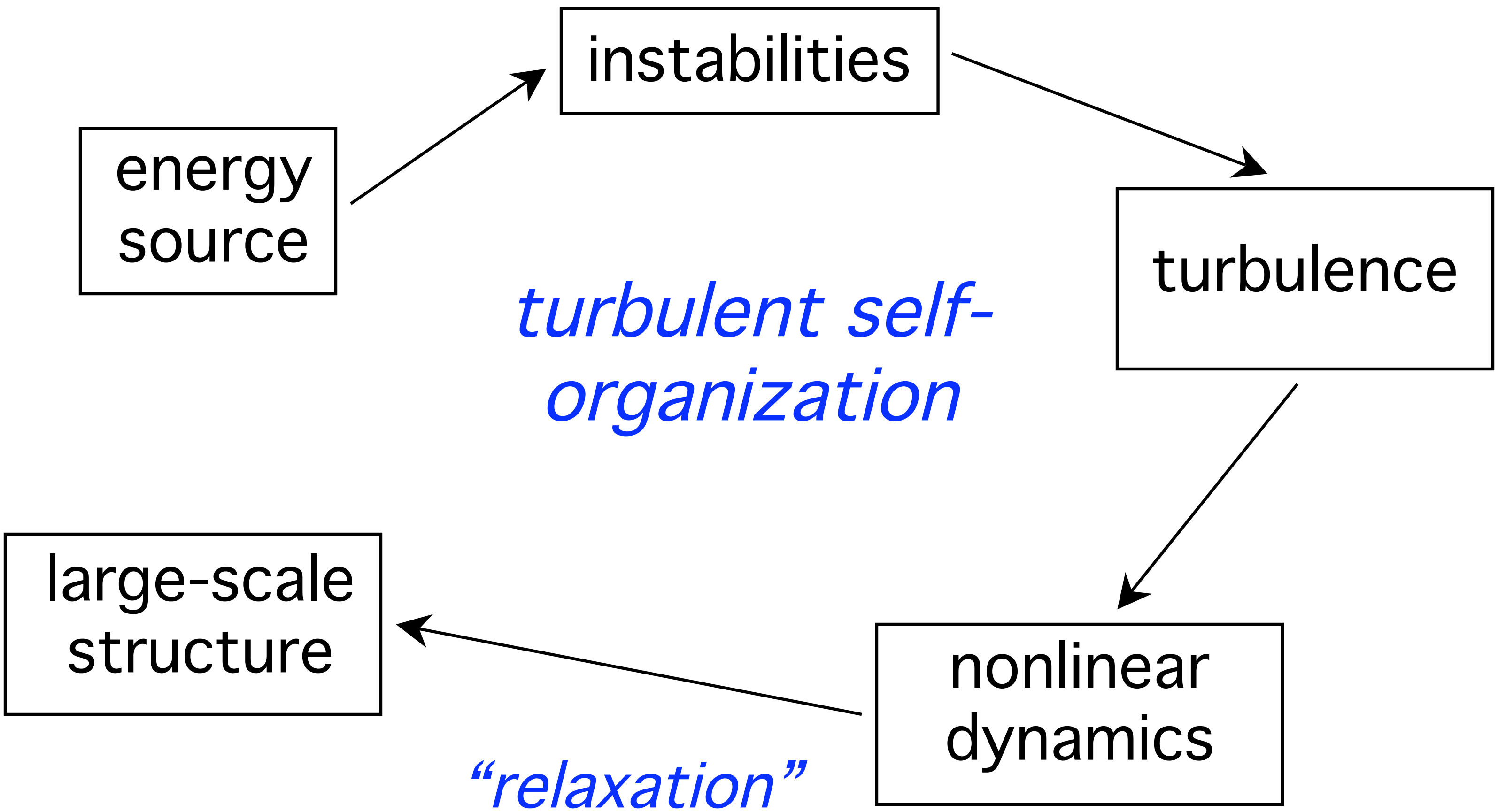
Outline

- Self-organization definition/examples
- Physics foundations
- Briefly: Driven Relaxation Experiment (DRX)

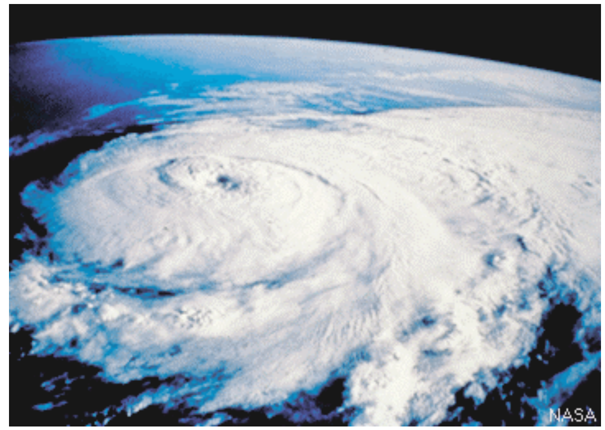
Self-organization is spontaneous generation of large-scale structure in a turbulent medium



Why study plasma self-organization?

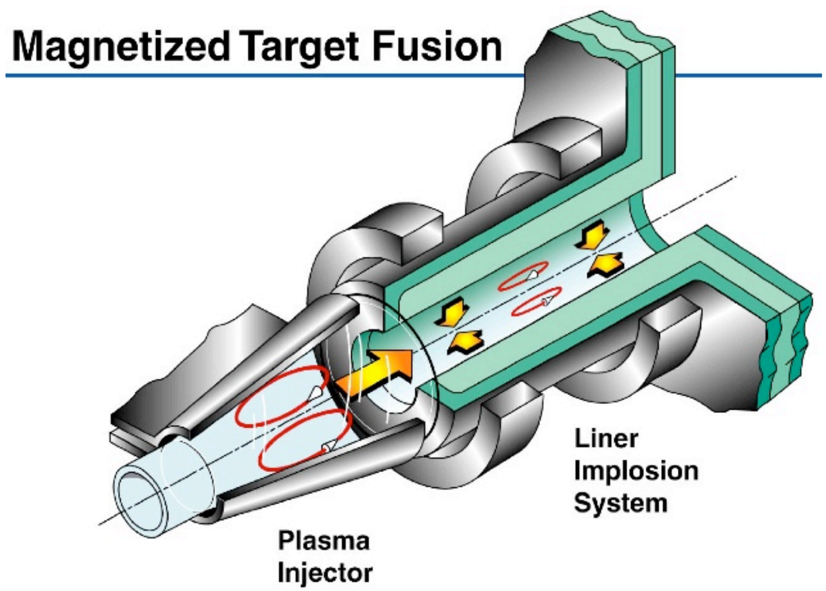
- Inspire an economic fusion reactor to help solve our energy problems
- Explain puzzles in solar, space, and astrophysical plasmas
- Fundamental to field of turbulence

Self-organization at work: Cyclonic and zonal flows in planetary atmospheres

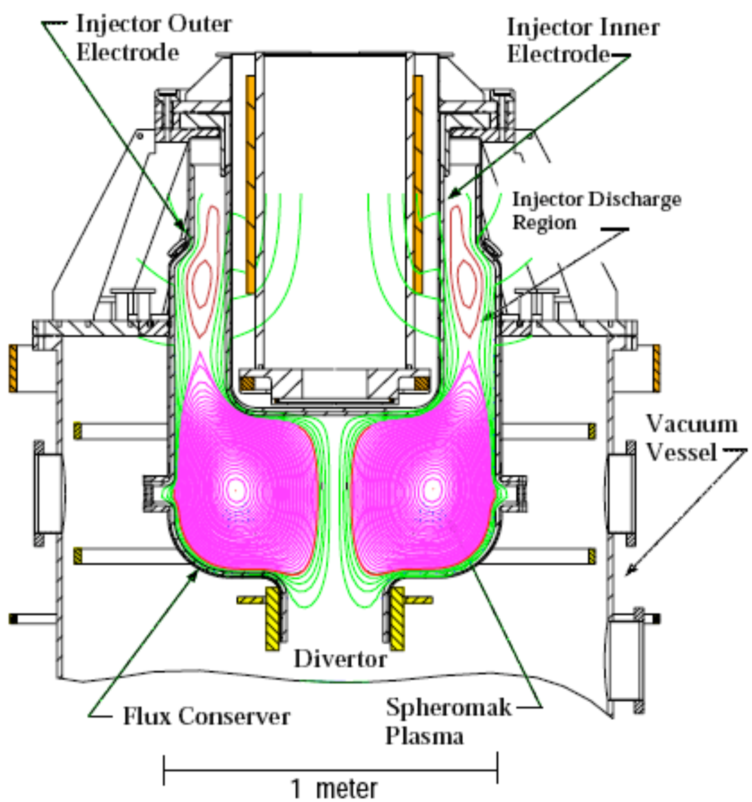


Self-organized plasmas are good candidates for alternate magnetic fusion energy concepts

Adiabatic compression of a field-reversed configuration (FRXL/MTF at LANL/AFRL):

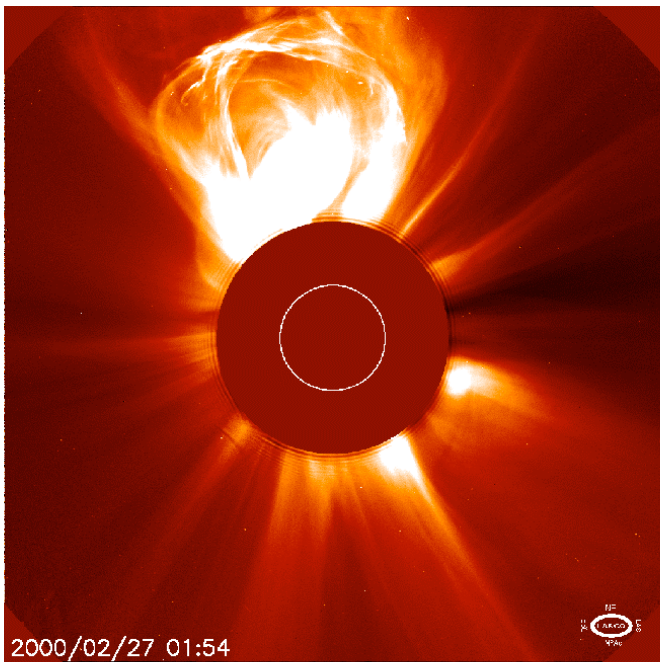


SSPX spheromak (at LLNL):

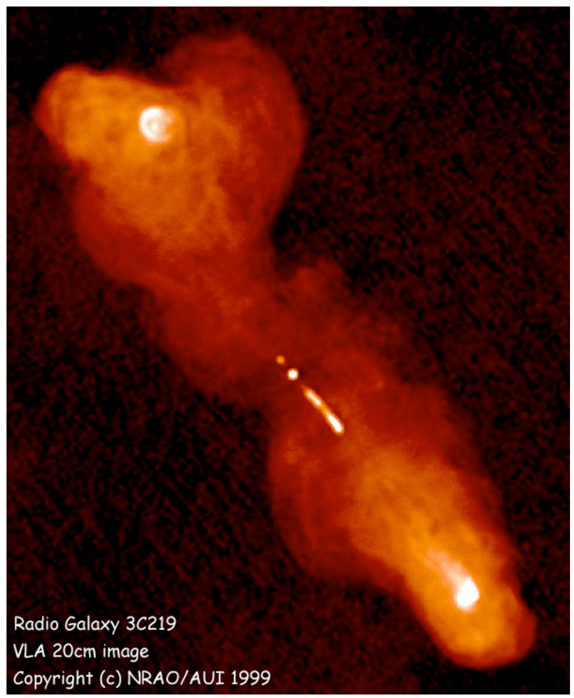


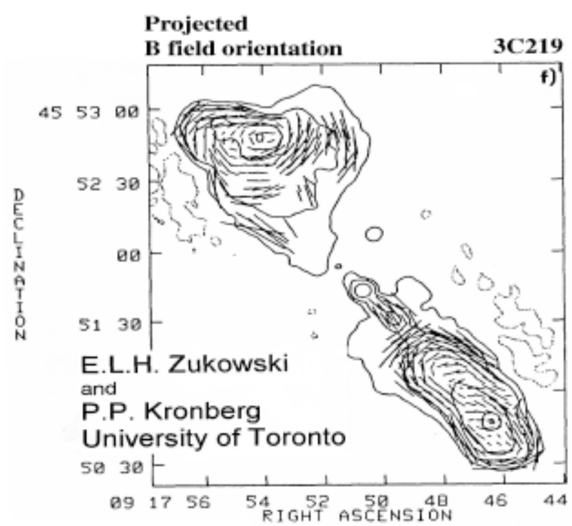
Self-organization may also explain cosmic magnetic structures on widely varying scales

coronal mass ejection (CME)



Extra-galactic jets/lobes





Self-organization is a consequence of three physical properties of turbulent systems

- "Ideal invariants"
- "Selective decay"
  - "Taylor relaxation"
- "Inverse spectral cascade"

Ideal invariants of a magnetohydrodynamic* (MHD) system

2D MHD:

energy $$E = \frac{1}{2}\int (v^2 + B^2)\, \mathrm{d}^2 x$$

cross-helicity $$H_C = \int \mathbf{v} \cdot \mathbf{B}\, \mathrm{d}^2 x$$

mean square magnetic potential $$A = \int \psi^2\, \mathrm{d}^2 x$$

3D MHD: integrals become volume integrals

A replaced by magnetic helicity $$H_M = \int \mathbf{A} \cdot \mathbf{B}\, \mathrm{d}V.$$

*come to July 13 lecture for MHD tutorial!

Ideal invariants will decay in presence of dissipation

3D MHD:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\eta \int j^2 \,\mathrm{d}V - \nu \int \omega^2 \,\mathrm{d}V,$$

$$\frac{\mathrm{d}H_C}{\mathrm{d}t} = -(\nu + \eta) \int \mathbf{j} \cdot \omega \,\mathrm{d}V,$$

$$\frac{\mathrm{d}H_M}{\mathrm{d}t} = -\eta \int \mathbf{j} \cdot \mathbf{B} \,\mathrm{d}V.$$

where η is resistivity, ν is kinematic viscosity, **j** is current density, **B** is magnetic field, and ω is vorticity

Ideal invariants decay at different rates ⇒ "selective decay"

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -\eta \int j^2 \,\mathrm{d}V - \nu \int \omega^2 \,\mathrm{d}V$$ ~$[B^2][L^3]$

$$\frac{\mathrm{d}H_M}{\mathrm{d}t} = -\eta \int \mathbf{j} \cdot \mathbf{B} \,\mathrm{d}V.$$ ~$[B^2][L^4]$

$$\frac{\mathrm{d}E}{\mathrm{d}t} \Big/ \frac{\mathrm{d}H_M}{\mathrm{d}t}$$ ~ L^{-1}

If L very small, then $|dE/dt| >> |dH_m/dt|$ ⇒ energy decays much more quickly than magnetic helicity!

Selective decay restricts system evolution to certain preferred states

If magnetic helicity decays very slowly compared to magnetic energy…

Model this process as variational problem (Woltjer, 1958; Taylor, 1974):

$$\delta\left(\int \frac{1}{2}(v^2 + B^2)\,\mathrm{d}V - \frac{1}{2}\lambda \int \mathbf{A}\cdot\mathbf{B}\,\mathrm{d}V\right) = 0$$

Insert $\mathbf{B} = \mathbf{B}_{me} + \delta\mathbf{B}$

Turn the crank and find $\mathbf{B}_{me}$ satisfies:

$$\nabla \times \mathbf{B} - \lambda \mathbf{B} = 0$$

$\Rightarrow$ **j** and **B** become aligned (“force-free”)

Wolter-Taylor relaxation

"Taylor relaxation" in a plasma leads to lowest (allowed) energy force-free state

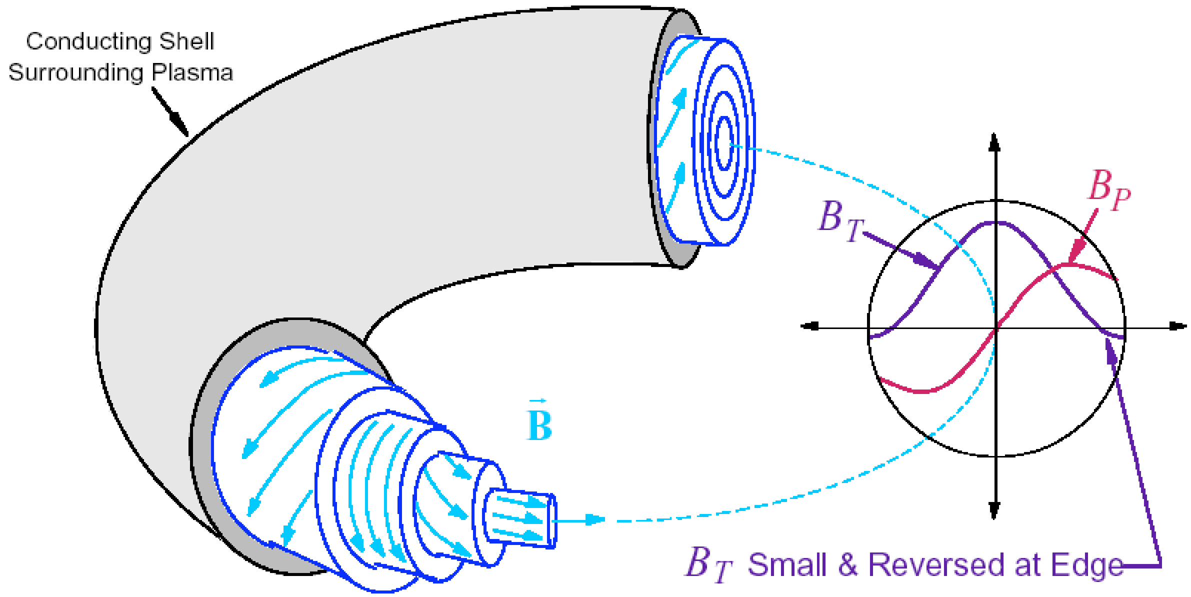
Taylor relaxed states have λ = global constant

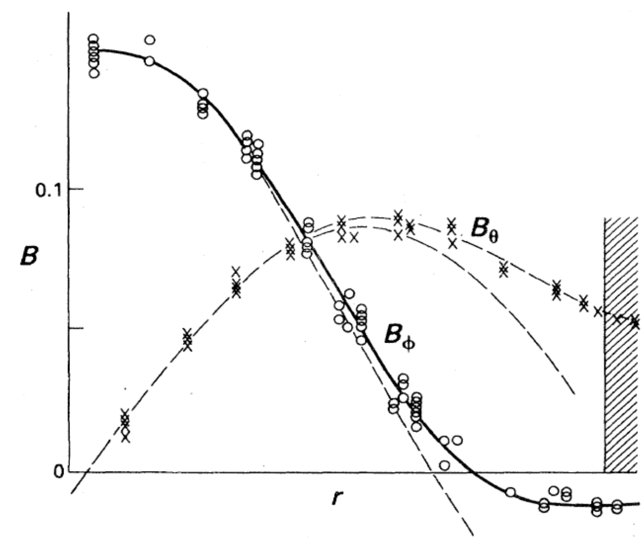
Solutions of $\nabla \times \mathbf{B} - \lambda \mathbf{B} = 0$
(in cylindrical coordinates) are Bessel functions (Lundquist, 1950):

$$B_z = B_0 \, J_0 \, (\lambda r)$$

$$B_\phi = B_0 \, J_1(\lambda r)$$

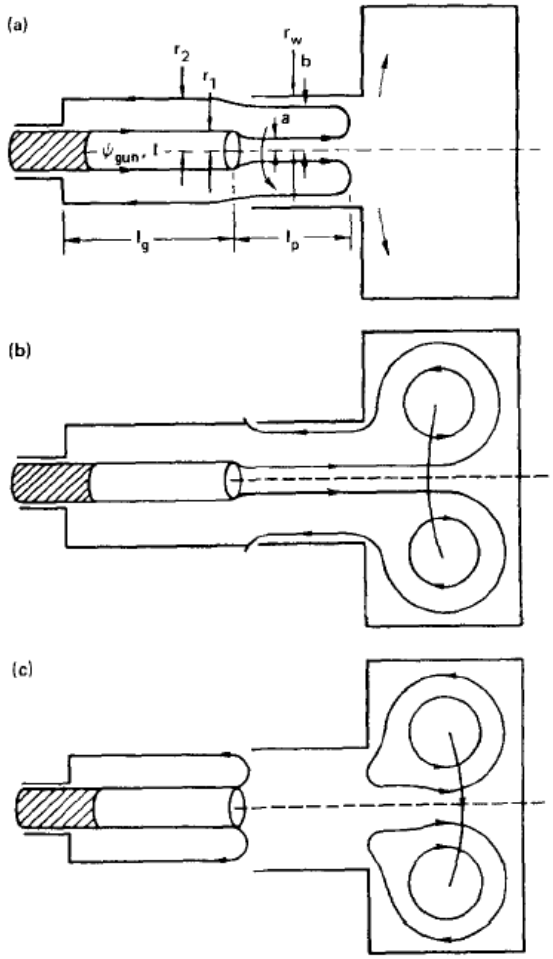
Reversed-field pinch (RFP) as a Taylor relaxed state

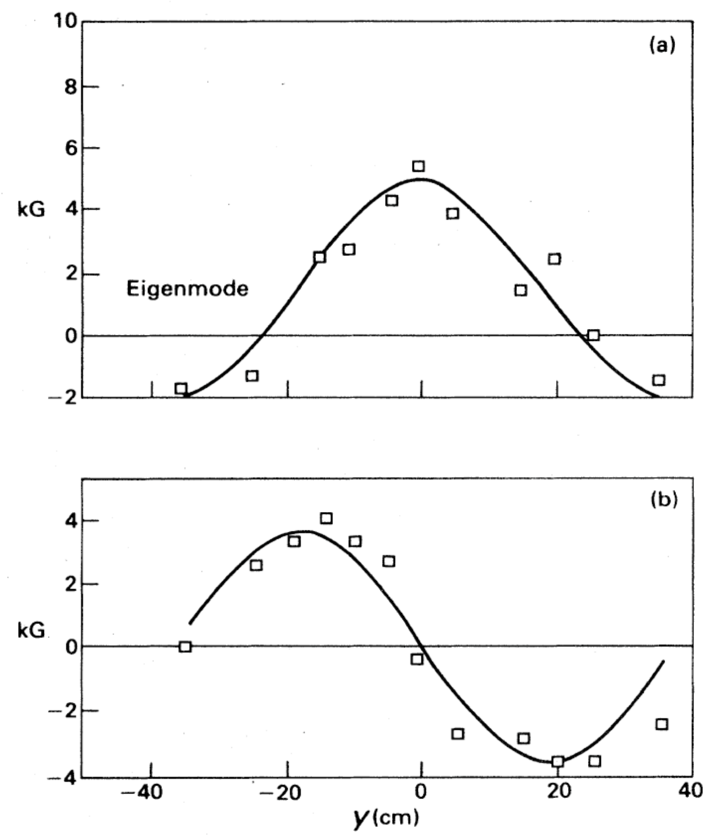




Bodin & Newton, *Nucl. Fus.*, 1980

Spheromak as a Taylor relaxed state

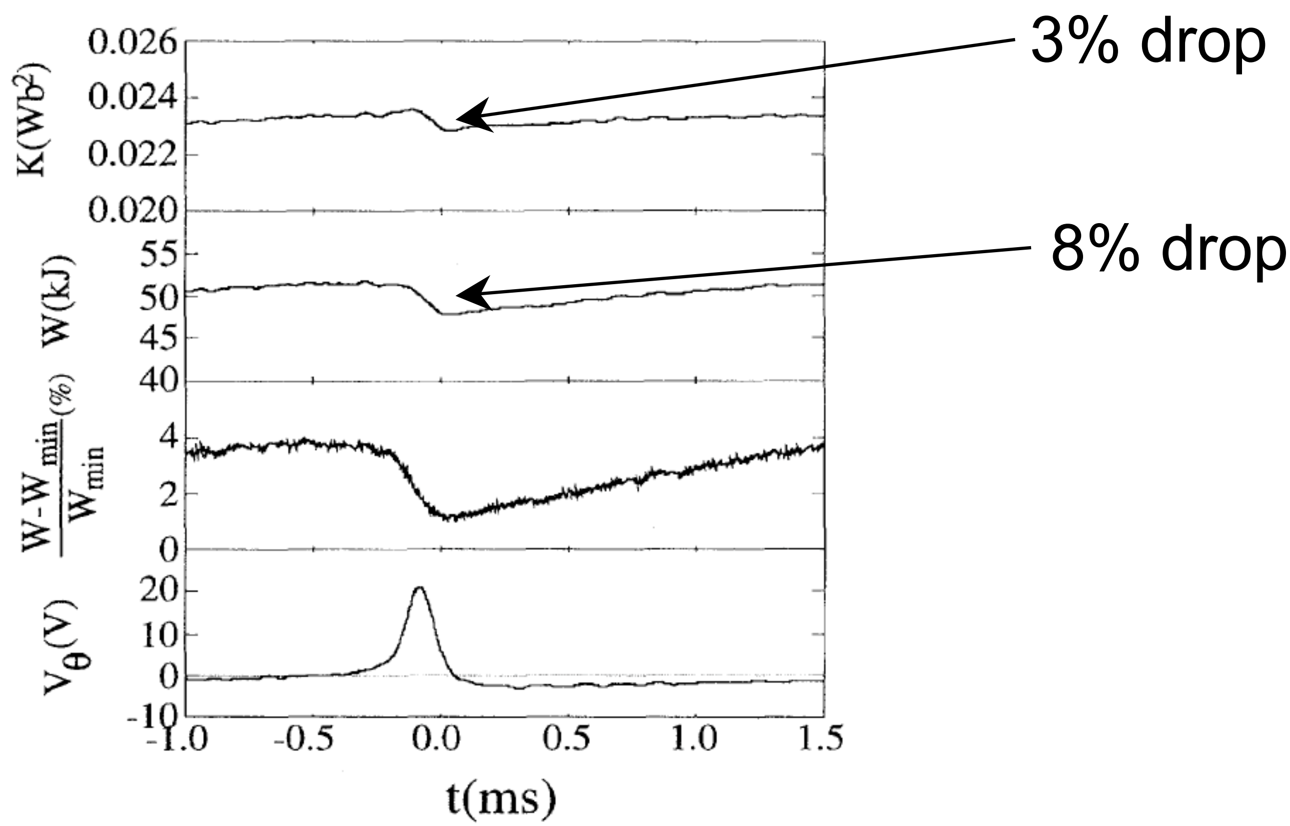




Turner et al., *Phys. Fluids.*, 1980

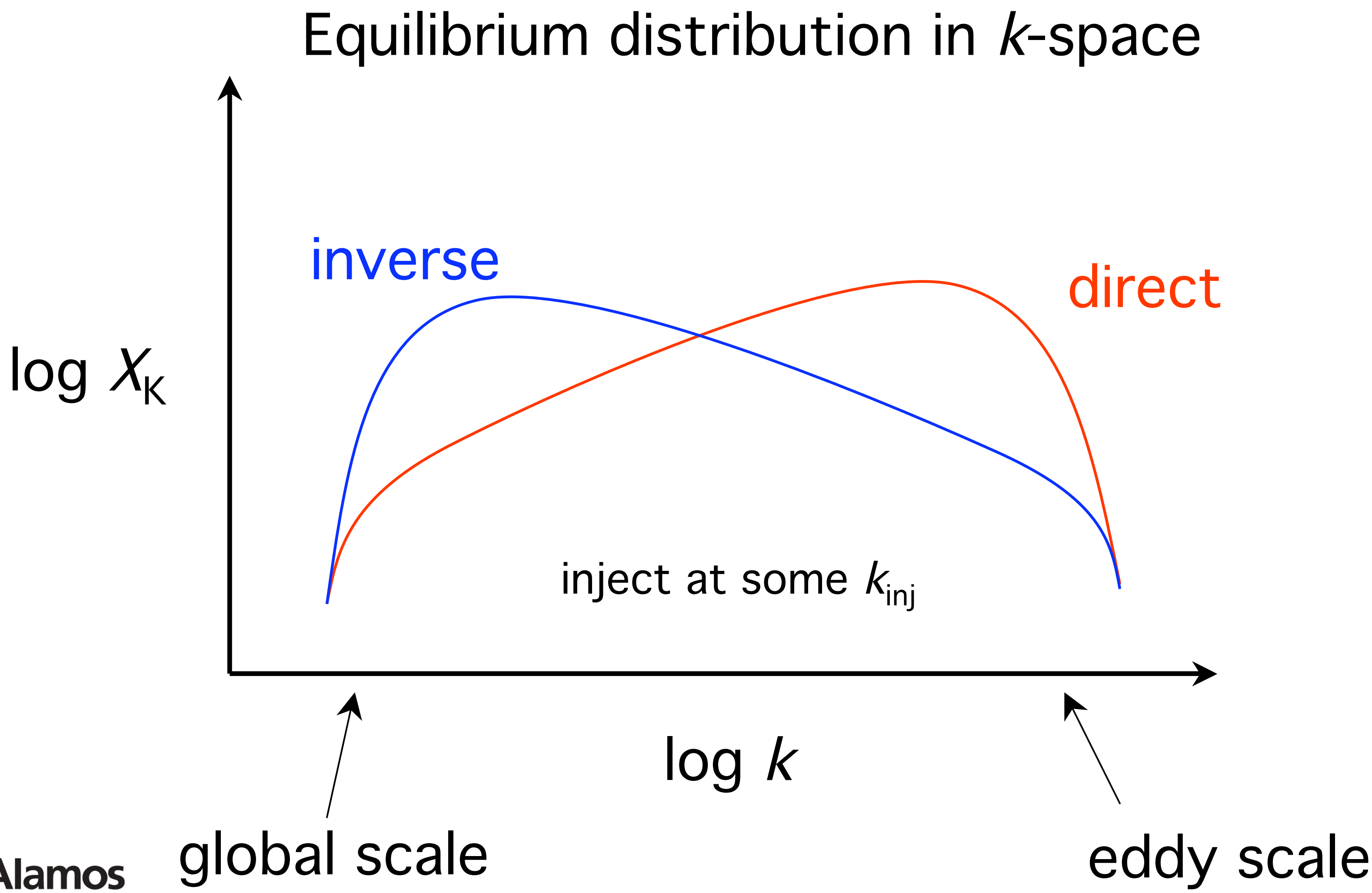
However, verification of magnetic helicity conservation during relaxation still elusive

Best results so far are from RFP relaxation events:



Ji et al., *PRL*, 1995

Transfer process of an ideal invariant in *k*-space is called a "cascade" in turbulence



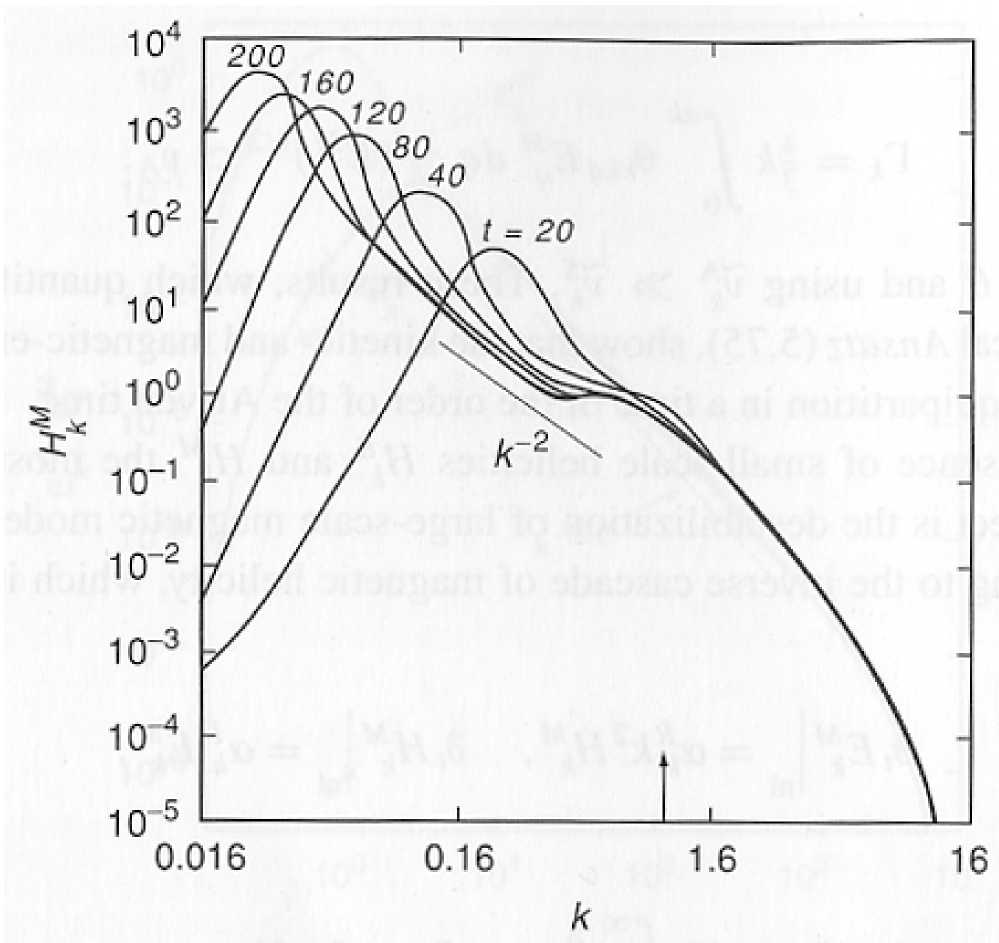
Cascade directions can be calculated or numerically computed but need to be measured

- Calculate Fourier spectra using equilibrium distribution in phase space given by Gibbs' potential
  $\rho_G = Z^{-1} \exp(-\alpha E - \beta H_M - \gamma H_C)$ (Frisch, 1975, *JFM*)

- Numerically solve via direct numerical simulation (DNS) (many people doing this now)

- Ultimately, spectral densities need to be measured in a laboratory experiment! (fame and glory guaranteed if you do this for your PhD dissertation)

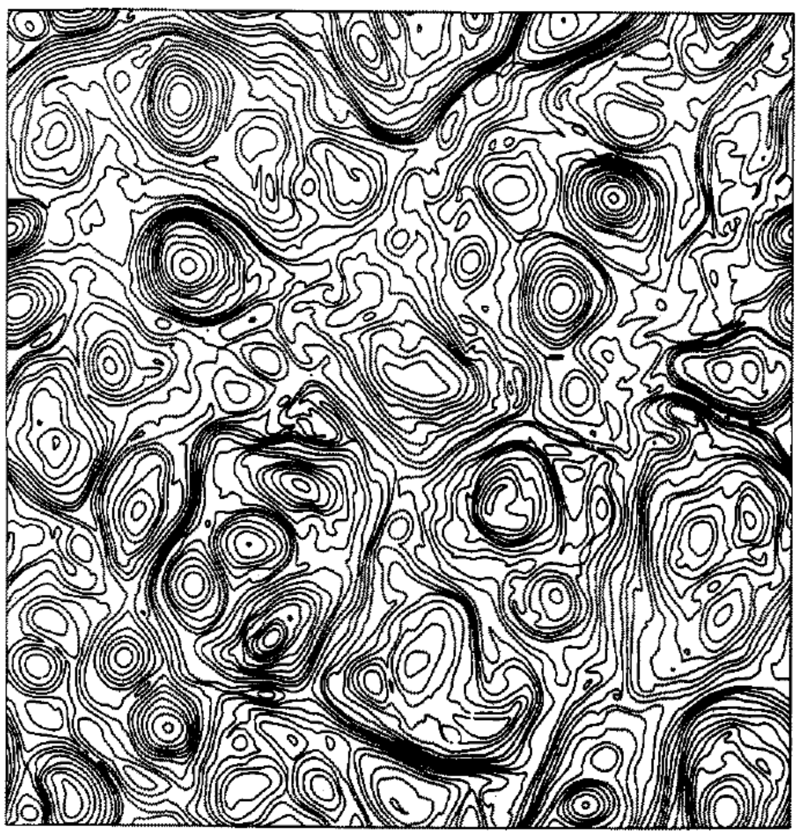
Numerical simulations for 3-D MHD turbulence show inverse cascade for magnetic helicity



Pouquet et al., 1976, *JFM*

A plot of magnetic potential in 2-D MHD turbulence illustrates effect of inverse cascade

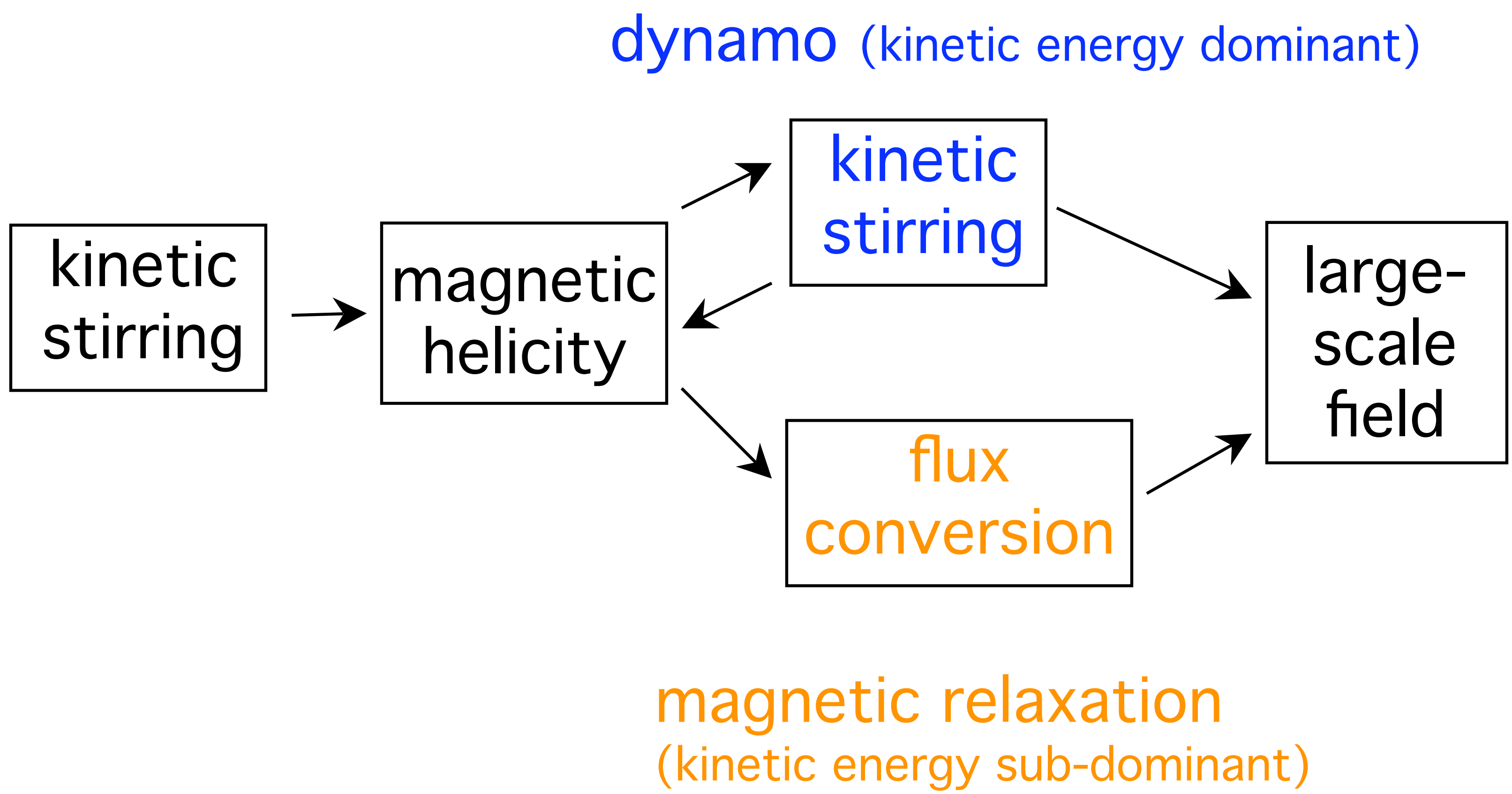
earlier time



later time

Biskamp & Welter, 1989, *PFB*

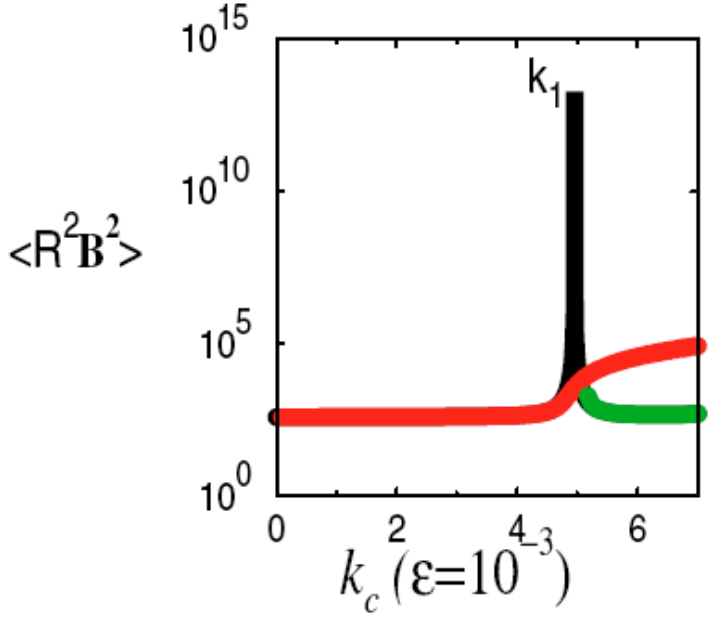
Aside: “Dynamo” is important related concept for origins of (extra-)galactic magnetic fields



Driven Relaxation Experiment (DRX) will try to create and sustain novel relaxed states

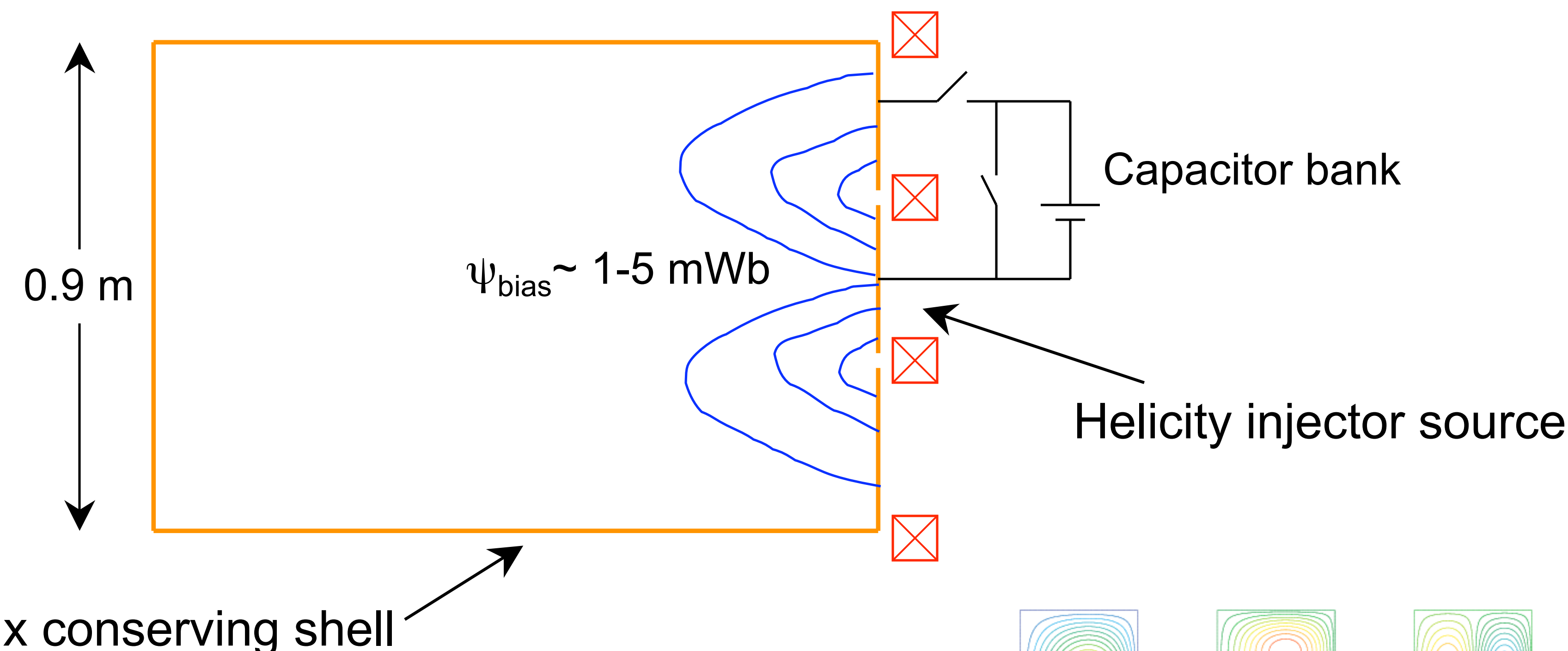
Physics goals of DRX:

- Determine accessibility of $\lambda \geq \lambda_1$ relaxed states; characterize them (large-scale structure)
- Measure wavenumber spectrum (cascade; dynamo or relaxation)
- Explore dynamics of relaxation (instabilities)

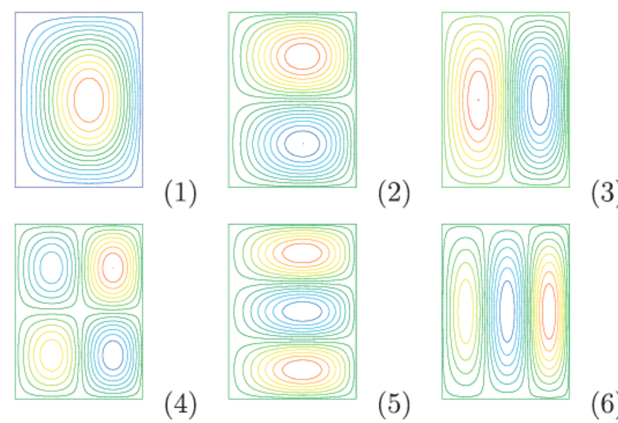


Tang & Boozer, PRL, 2005

DRX conceptual design



Higher order relaxed states



Tang & Boozer, 2005, *PRL*

Summary

- Plasma self-organization is a frontier research area in plasma physics
  - Innovative fusion configurations
  - Structure of extra-galactic magnetic fields
  - Fundamental to turbulence studies
- Three key properties of turbulence lead to self-organization
  - Ideal invariants
  - Selective decay
  - Inverse cascade
- DRX, a new LANL experiment furthering studies of magnetic relaxation

If you are interested in learning more…

- Reference texts
  - D. Biskamp, *Nonlinear Magnetohydrodynamics*
  - D. Biskamp, *Magnetohydrodynamic Turbulence*
  - P. Bellan, *Spheromaks*
  - U. Frisch, *Turbulence*
- Website of NSF Frontier Center on Magnetic Self Organization in Laboratory and Astrophysical Plasmas: http://www.cmso.info
- Some graduate programs with opportunities in this area:
  - Princeton (astrophysical sciences, program in plasma physics)
  - Univ. of Wisconsin-Madison (physics)
  - UCLA (physics & astronomy)
  - Caltech (applied physics)
  - Univ. of Washington (aeronautics & astronautics)
- Or contact scotthsu@lanl.gov