A Gentle Introduction to Magnetic Reconnection

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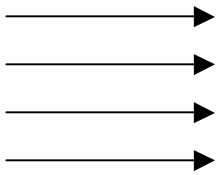
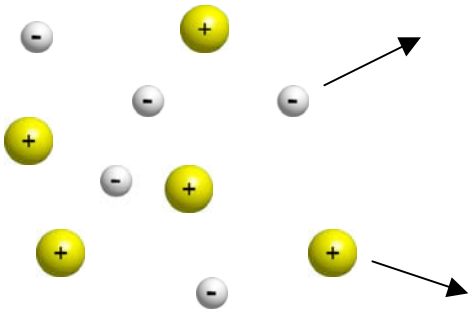
Presented at:
Los Alamos Plasma Physics Summer School
July 25, 2006

What is Magnetic Reconnection?

Magnetic Energy → **Kinetic Energy**



$$\frac{B^2}{8\pi}$$



$$\frac{mV^2}{2}$$

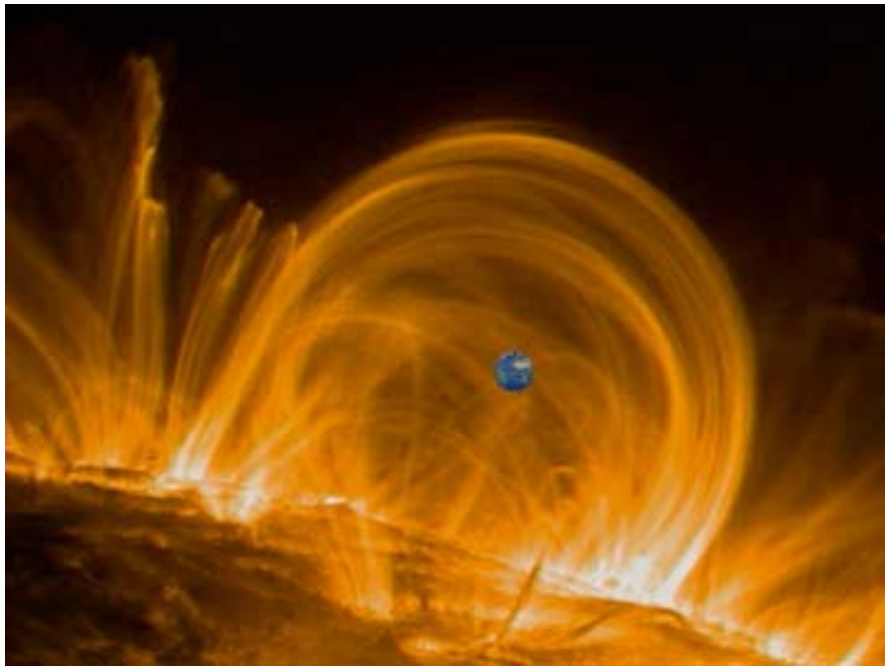
Many applications →

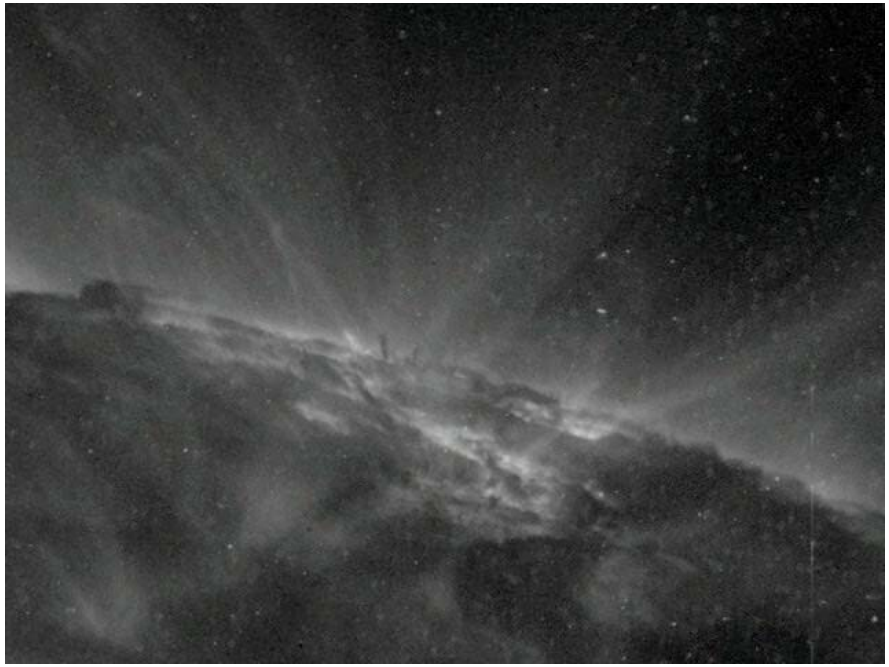
Space Plasmas

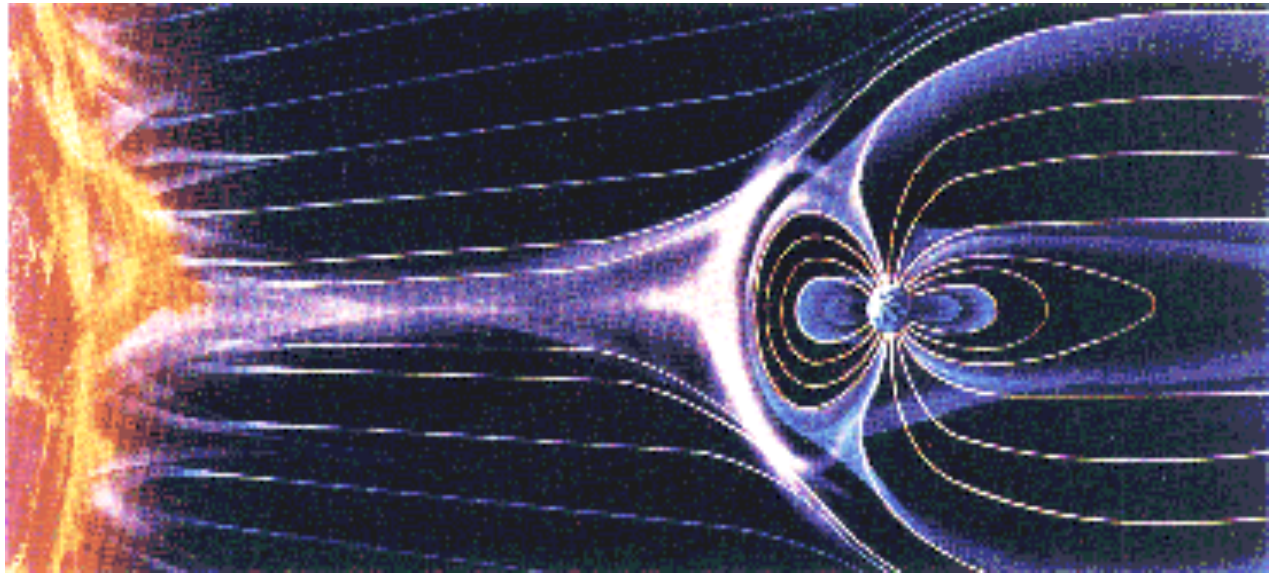
Laboratory Experiments

Astrophysical Plasmas

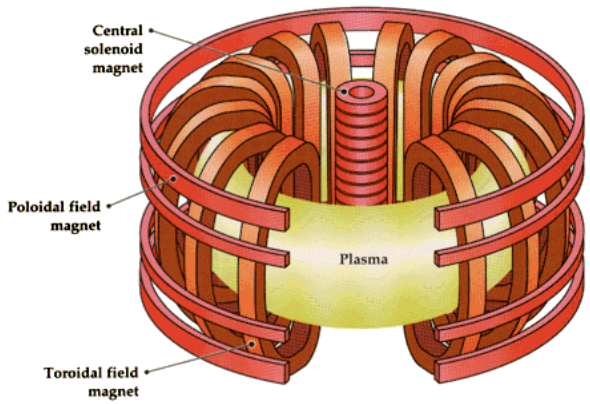
Reconnection in Space Plasmas

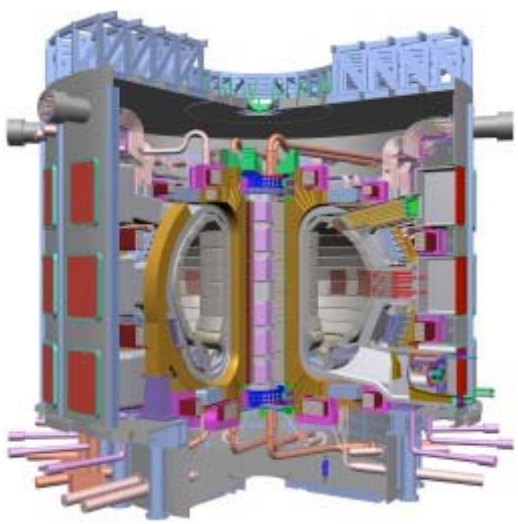


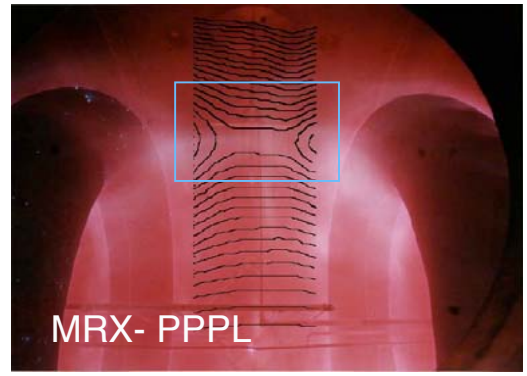


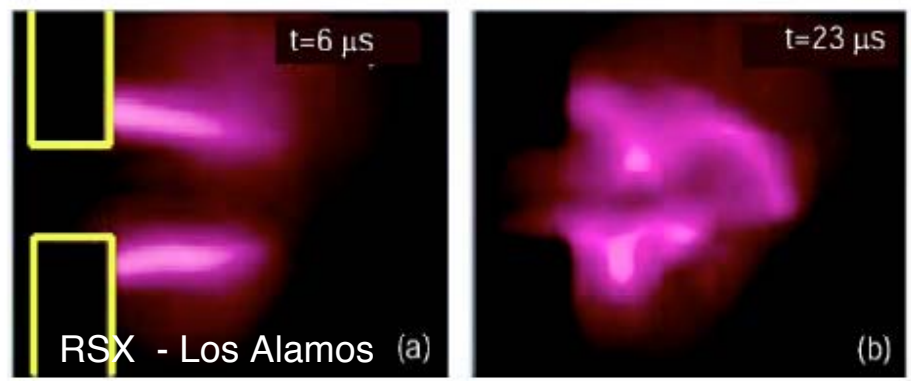


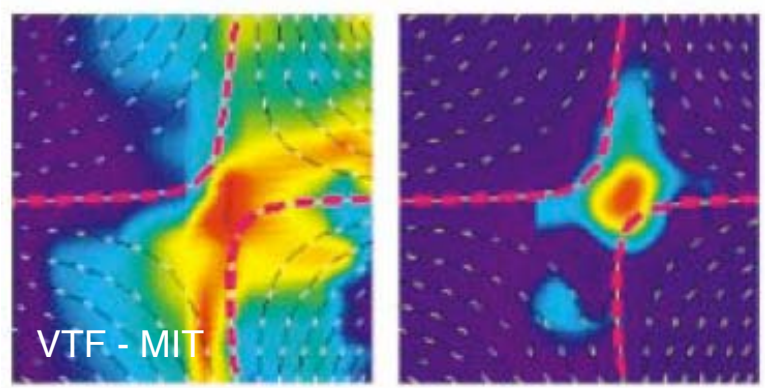
Reconnection in Laboratory Plasmas





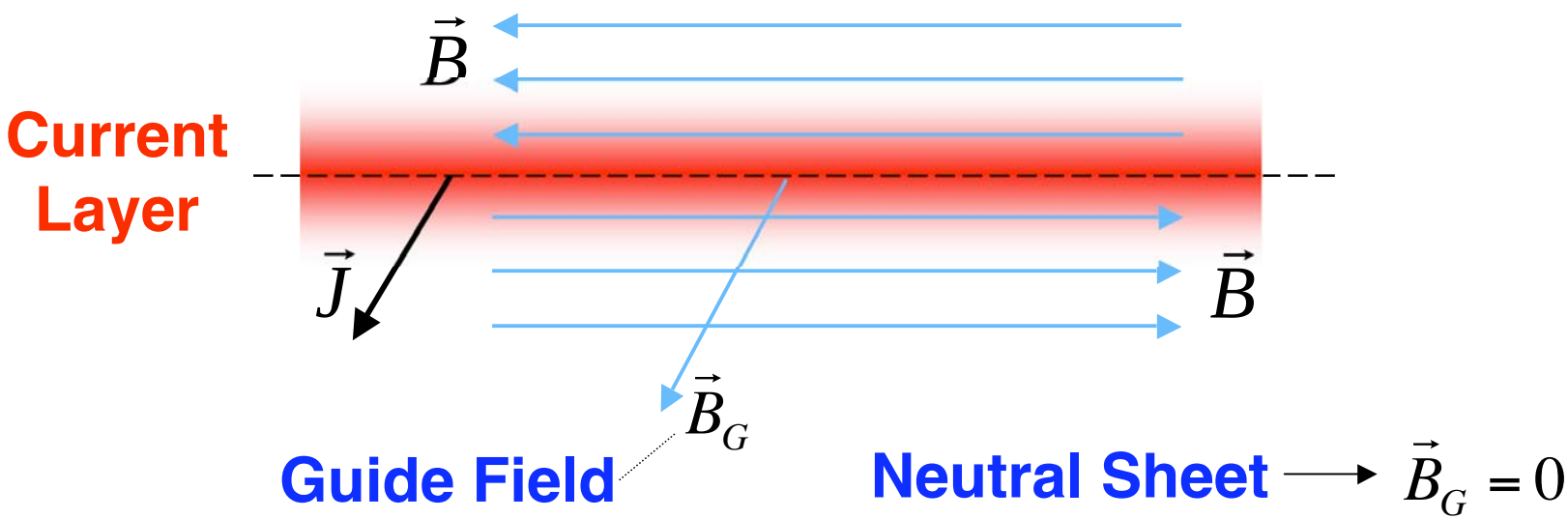






What is a Current Sheet?

Current layer + corresponding field reversal

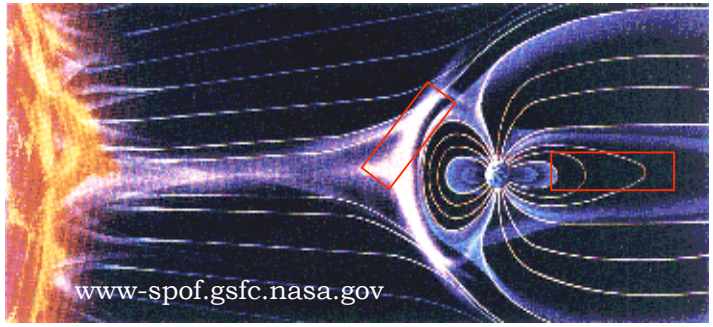


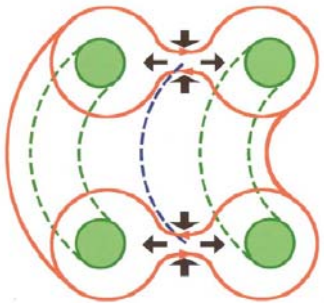
SciSearch Database
"current + sheet"

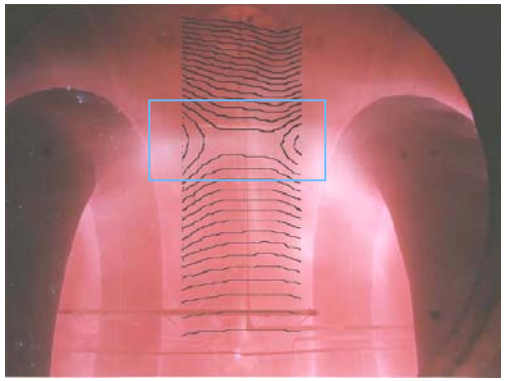
~ 10,000 papers
since 1960

Examples of Current Sheets

- Magnetotail
- Magnetopause
- Heliospheric current sheet
- Plasma tail of comets
- Solar flares & prominences
- Simple geometry to study magnetic reconnection
- Laboratory plasmas – MRX experiment at PPPL

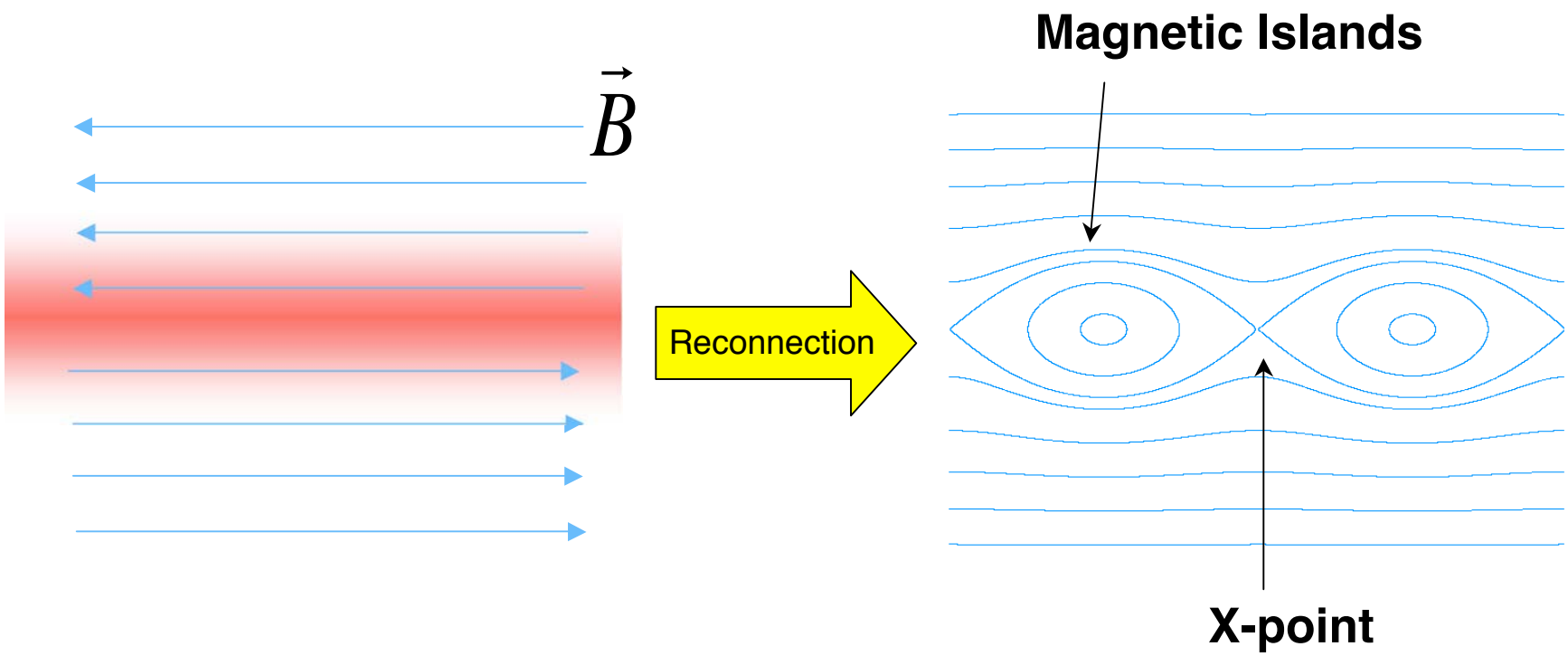




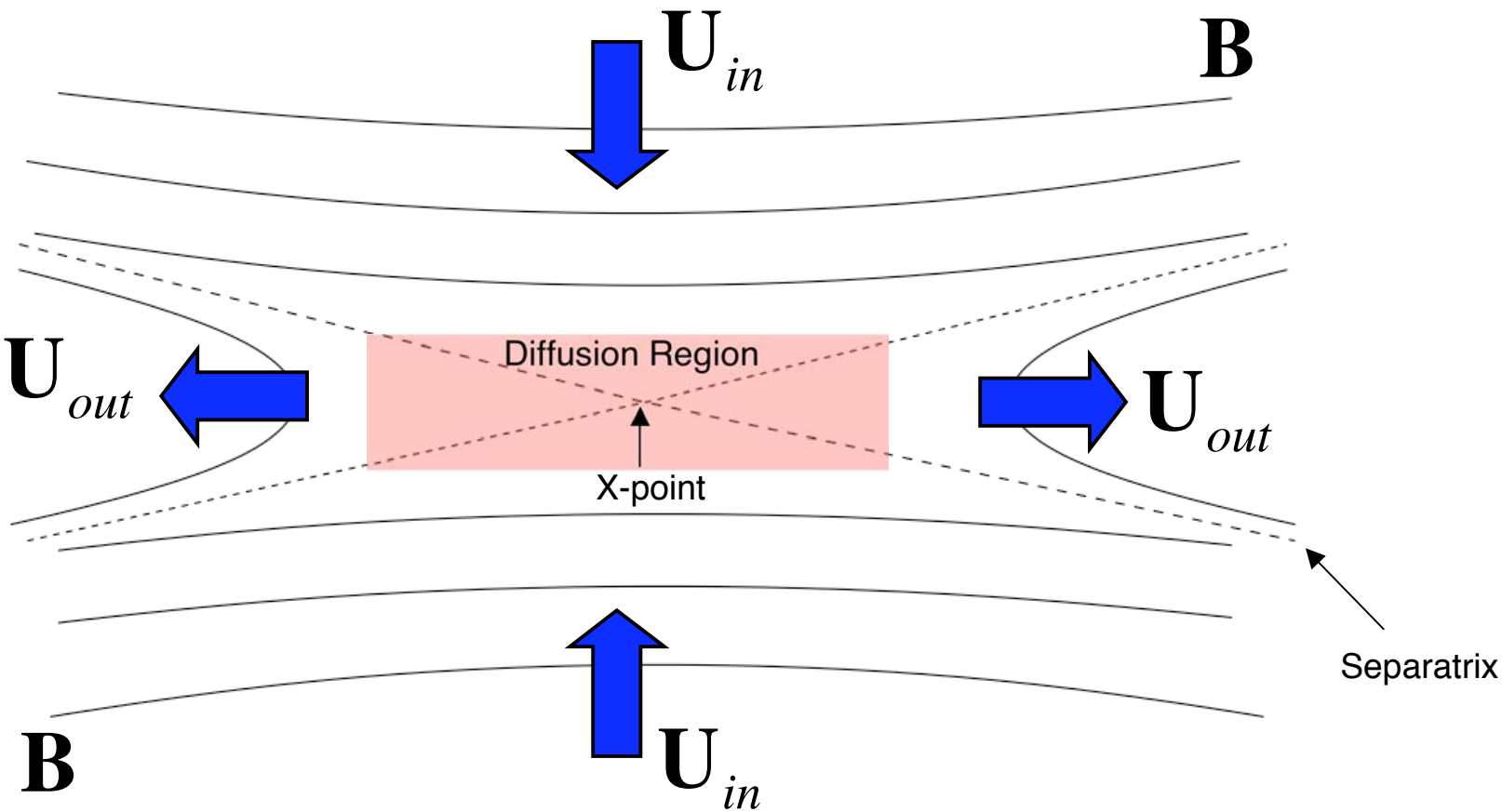


Figures courtesy of Hantao Ji (PPPL)

Topological Consequences



More Terminology



Collisional vs Collisionless Reconnection

$$\nu \propto \frac{n}{T^{3/2}}$$

Magnetotail Parameters

$T_e \sim 1$ keV $\quad T_i \sim 6$ keV

$n \sim 1$ cm^{-3} $\quad B \sim 20$ nT

L ~ 1000 km

$\longrightarrow$

$\nu_e \sim 10^{-9}$ sec^{-1} $\quad \Omega_{ci} \sim 1$ sec^{-1}

$\frac{V_{the}}{\nu_e L} \sim 10^{10}$ $\quad \frac{\rho_i}{L} \sim 1$

Questions for Collisionless Regime:

1. **How does reconnection proceed so rapidly in collisionless regimes?**

2. **How does it get started in the first place?** $\Rightarrow$ **Onset problem**

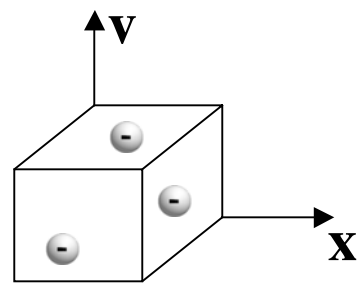
What equations describe a plasma?

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \bullet \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s}\left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) \bullet \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

Vlasov

$$\varepsilon = 0$$

$$f_s(\mathbf{x}, \mathbf{v}, t) \longrightarrow \frac{\text{Number of particles}}{\text{Unit volume of phase space}}$$



Small Parameter $\longrightarrow$ $\varepsilon = \dfrac{1}{n\lambda_D^3} \sim 10^{-6} \rightarrow 10^{-12}$

Maxwell's Equations

$$\rho = \sum_s q_s \int f_s \, d\mathbf{v}$$

$$\mathbf{J} = \sum_s q_s \int \mathbf{v} \, f_s \, d\mathbf{v}$$

$$\nabla \bullet \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \bullet \mathbf{E} = 4\pi\rho \qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

Vlasov-Maxwell Theory

Vlasov

$$\frac{\partial f_s}{\partial t} + \mathbf{v} \bullet \frac{\partial f_s}{\partial \mathbf{x}} + \frac{q_s}{m_s}\left(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}\right) \bullet \frac{\partial f_s}{\partial \mathbf{v}} = 0$$

Maxwell

$$\nabla \bullet \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}$$

$$\nabla \bullet \mathbf{E} = 4\pi\rho \qquad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$$

- **Coupled by first 2 moments** $\rho = \sum_s q_s \int f_s \, d\mathbf{v}$ $\quad \mathbf{J} = \sum_s q_s \int \mathbf{v} \, f_s \, d\mathbf{v}$
- **Complete description of collisionless plasma**
- **Very difficult to solve - 6D phase space!**
- **Fluid description is much easier**

Fluid Description of Plasma

Density $\longrightarrow$ $n_s = \int f_s \, d\mathbf{v}$

Velocity $\longrightarrow$ $\mathbf{U}_s = \int \mathbf{v} \, f_s \, d\mathbf{v}$

Pressure $\longrightarrow$ $\mathbf{P}_s = m_s \int (\mathbf{v} - \mathbf{U}_s)(\mathbf{v} - \mathbf{U}_s) f_s \, d\mathbf{v}$

Take velocity space moments of the Vlasov Equation:

Mass conservation $\longrightarrow$ $$\frac{\partial n_s}{\partial t} + \nabla \bullet (n_s \mathbf{U}_s) = 0$$

Momentum conservation $\longrightarrow$ $$m_s n_s \frac{d\mathbf{U}_s}{dt} = -\nabla \bullet \mathbf{P}_s + q_s n_s \left(\mathbf{E} + \frac{\mathbf{U}_s \times \mathbf{B}}{c} \right)$$

Closure Problem - Each equation contains higher order moment!

MHD Model

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{U}) = 0 \quad \longrightarrow \quad \textbf{Mass conservation}$$

$$\rho \frac{d\mathbf{U}}{dt} = -\nabla \mathbf{P} + \frac{\mathbf{J} \times \mathbf{B}}{c} \quad \longrightarrow \quad \textbf{Momentum conservation}$$

$$\mathbf{E} + \frac{\mathbf{U} \times \mathbf{B}}{c} = \eta \mathbf{J} \quad \longrightarrow \quad \textbf{Ohm's Law}$$

$$\frac{d}{dt}\left(\frac{P}{\rho^{\gamma}}\right) = 0 \quad \longrightarrow \quad \textbf{Adiabatic equation of state}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} \quad \longrightarrow \quad \textbf{Ampere's Law}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad \longrightarrow \quad \textbf{Faraday's Law}$$

$$\nabla \bullet \mathbf{B} = 0$$

Ideal MHD $\longrightarrow \eta = 0 \longrightarrow \mathbf{U}_{\perp} = c \dfrac{\mathbf{E} \times \mathbf{B}}{B^2}$

Evolution Equation for B Field

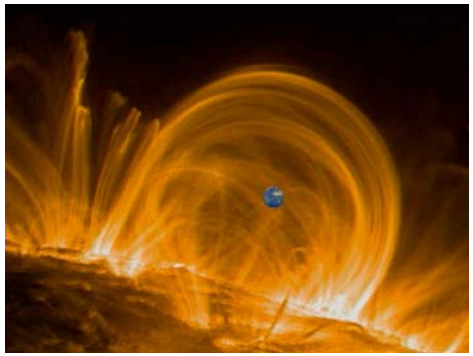
$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{\nabla \times \left(\mathbf{U} \times \mathbf{B}\right)}_{\text{Convection}} + \underbrace{\frac{\eta c^2}{4\pi} \nabla^2 \mathbf{B}}_{\text{Diffusion}}$$

Magnetic Diffusion Coefficient

$$D_M = \frac{\eta c^2}{4\pi} \sim \frac{(\text{Length})^2}{\text{time}}$$

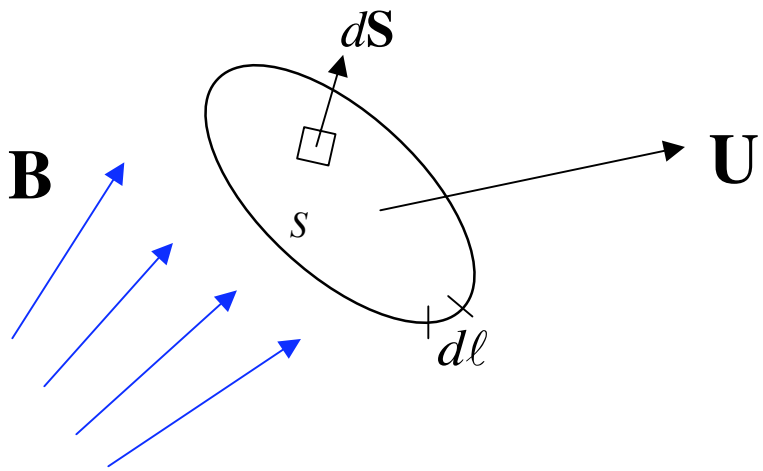
Time Scale for Resistive Diffusion

$$\tau_R = \frac{L^2}{D_M}$$



$\tau_R \sim 10{,}000$ years!

Frozen-in Condition



Magnetic Flux

$$\psi = \int_s \mathbf{B} \bullet d\mathbf{S}$$

$$\frac{d\psi}{dt} = \int_s \frac{\partial \mathbf{B}}{\partial t} \bullet d\mathbf{S} \quad - \oint (\mathbf{U} \times \mathbf{B}) \bullet d\ell$$

$$\frac{d\psi}{dt} = -c \oint \left(\mathbf{E} + \frac{\mathbf{U} \times \mathbf{B}}{c} \right) \bullet d\ell$$

Ohm's Law

$$\mathbf{E} + \frac{\mathbf{U} \times \mathbf{B}}{c} = \eta \mathbf{J}$$

Ideal MHD $\longrightarrow \dfrac{d\psi}{dt} = 0$

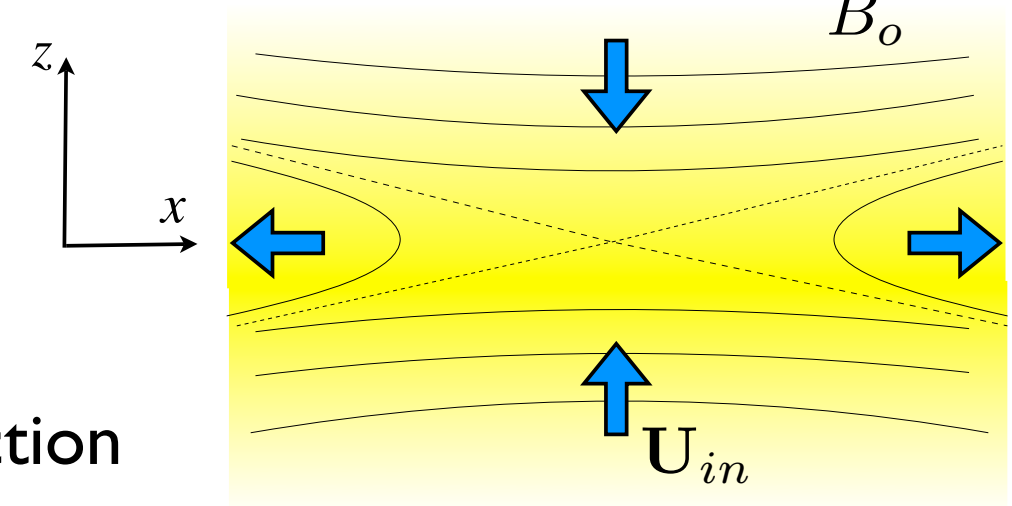
1. **Magnetic field is "frozen" into plasma**
2. **No Topological changes**

Define Basic Terms

$$\mathbf{U}_{in} = \frac{cE_y}{B_o}$$

- Reconnection rate

$$E_y = -\frac{1}{c}\frac{\partial A_y}{\partial t}$$



- Steady-state reconnection

$$\frac{\partial \mathbf{B}}{\partial t} = \frac{\partial \mathbf{E}}{\partial t} = 0 \longrightarrow E_y \text{ is spatially uniform in 2D}$$

- Collisionless reconnection

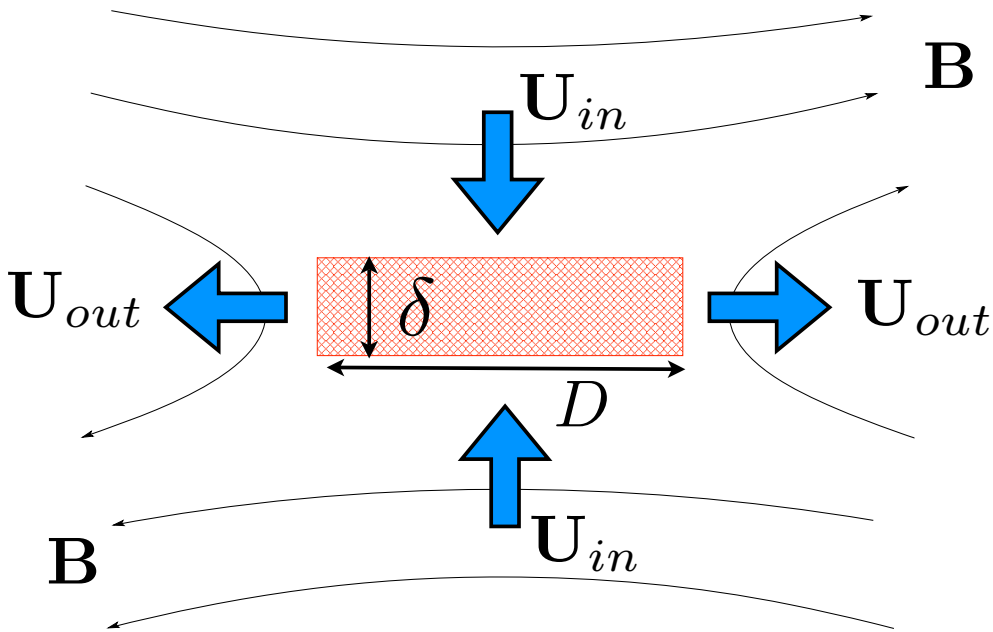
Dimensionless Reconnection Rate

$$E_R = \frac{\mathbf{U}_{in}}{V_A} = \frac{cE_y}{B_o V_A}$$

- Fast reconnection

Sweet and Parker's Model of Reconnection

1957 - 1958



$$\mathbf{U}_{out} = V_A = \frac{B_o}{\sqrt{4\pi\rho}}$$

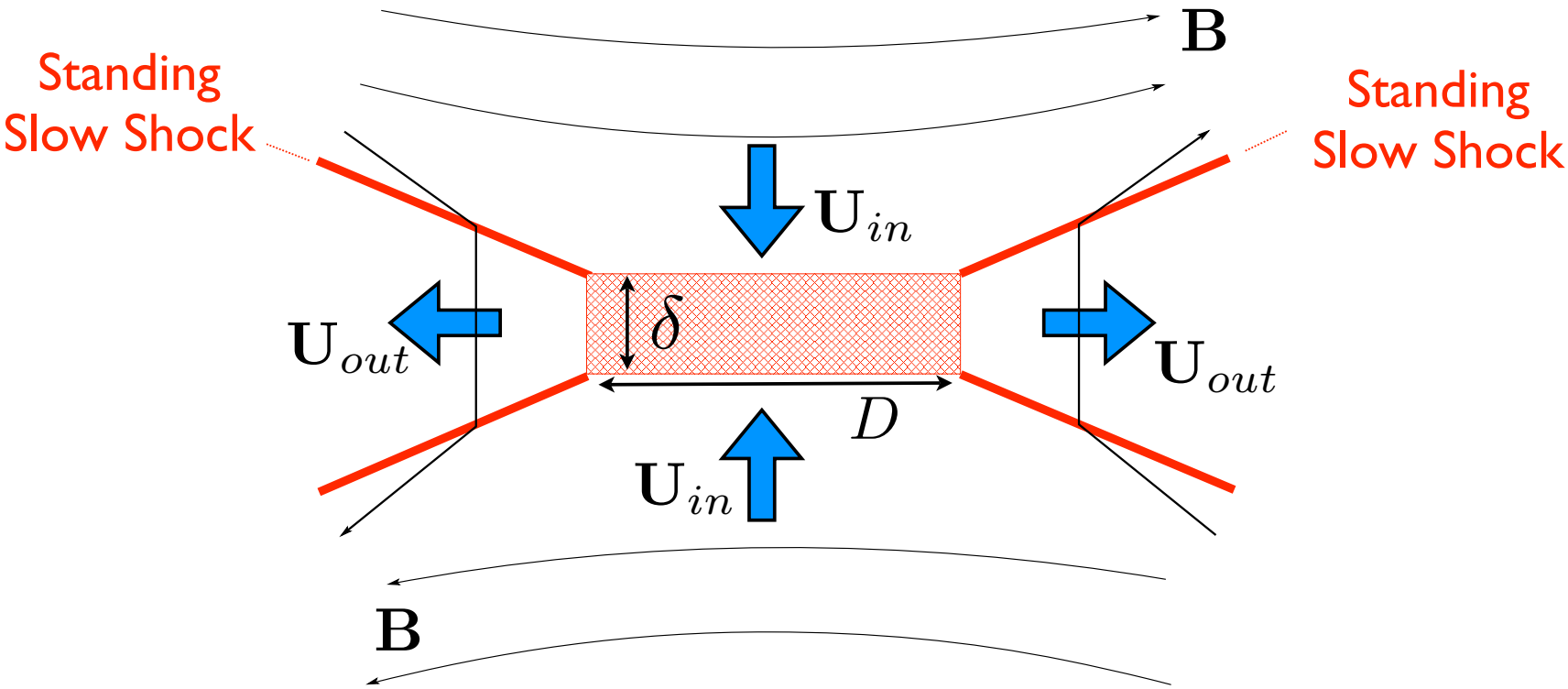
Reconnection Rate

$$\frac{\mathbf{U}_{in}}{V_A} = \frac{\delta}{D} = \frac{1}{\sqrt{S}}$$

$$S \equiv \frac{4\pi V_A D}{\eta c^2} = \frac{\tau_R}{\tau_A}$$

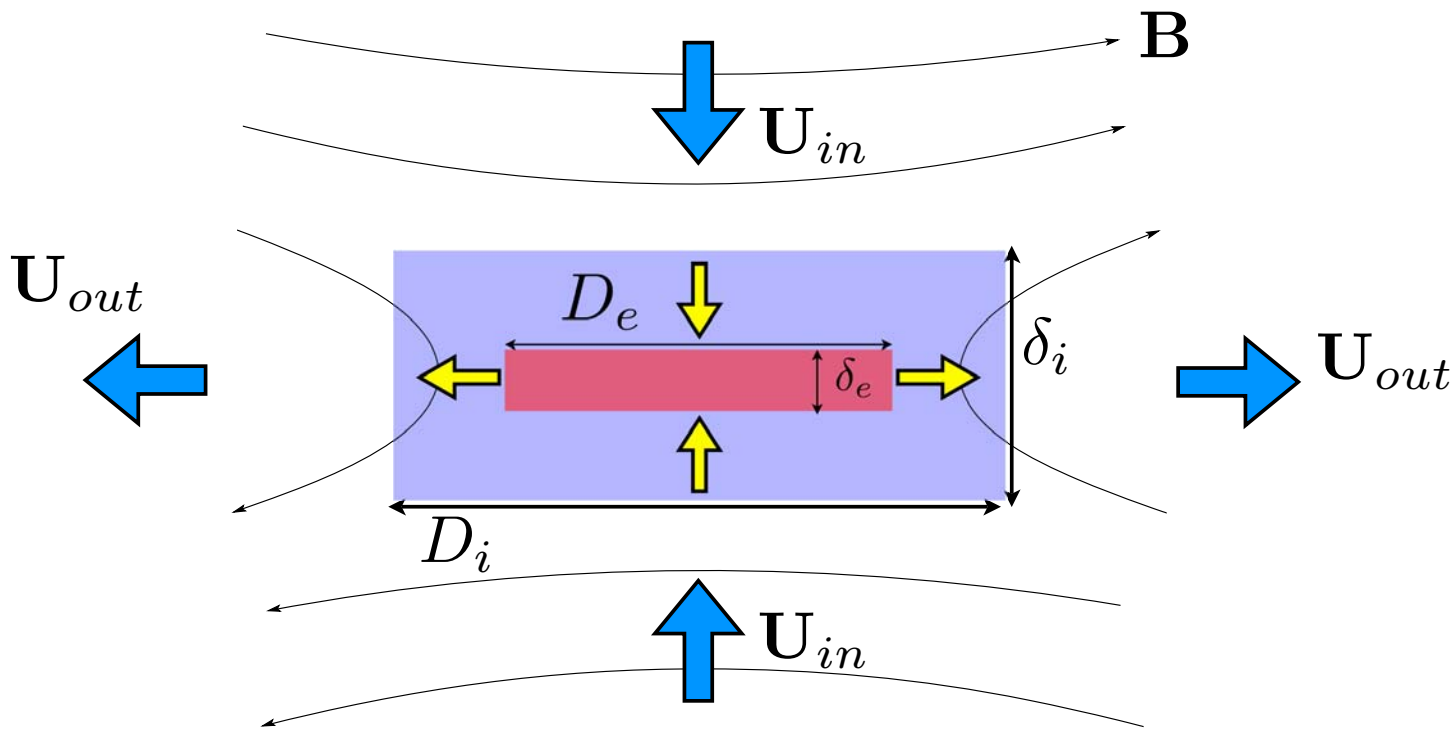
Lundquist Number

Petschek's Model - 1964



Reconnection Rate $\frac{\mathbf{U}_{in}}{V_A} \sim \frac{1}{\log(S)}$ $\longrightarrow$ Consistent with observations
Widely accepted for 20+ years!

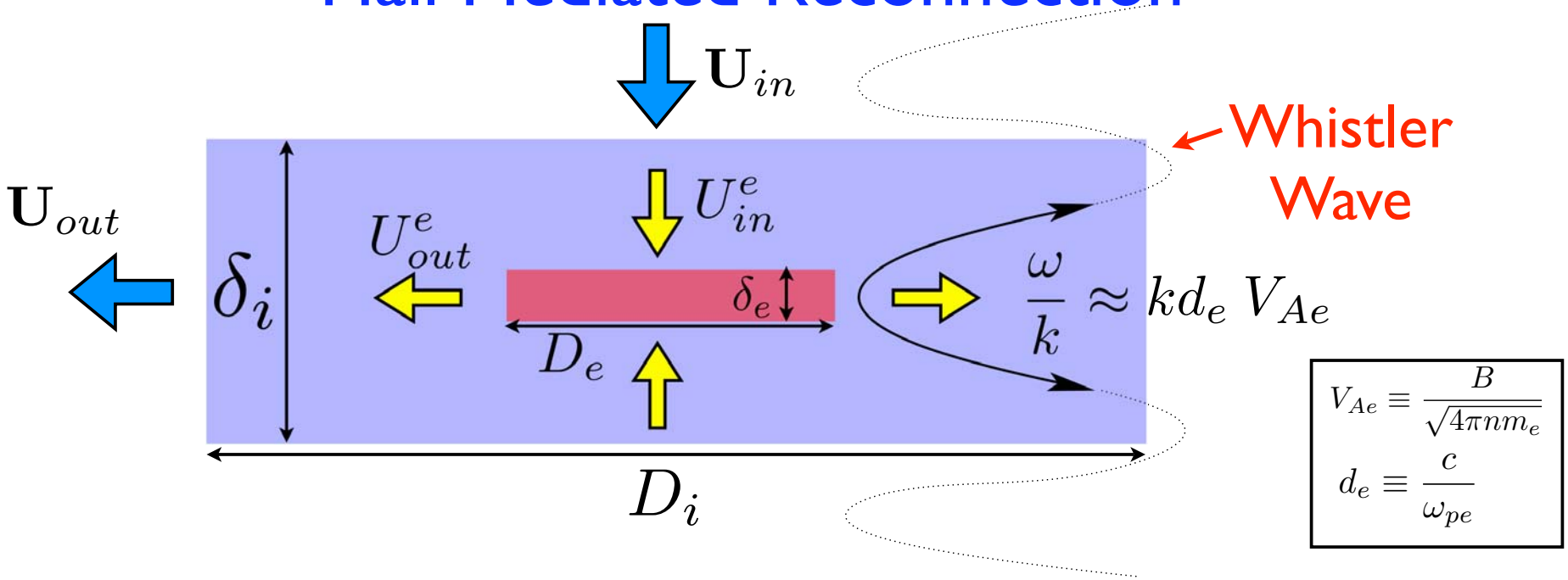
Hall Mediated Fast Reconnection 1995-2001



$\mathbf{E} + \frac{1}{c}\mathbf{U_i} \times \mathbf{B} \neq 0$ Frozen-in condition violated for ions

$\mathbf{E} + \frac{1}{c}\mathbf{U_e} \times \mathbf{B} \neq 0$ Frozen-in condition violated for electrons

Hall Mediated Reconnection



$$V_{Ae} \equiv \frac{B}{\sqrt{4\pi n m_e}}$$

$$d_e \equiv \frac{c}{\omega_{pe}}$$

Electron Diffusion Region

$$D_e U^e_{in} = \delta_e U^e_{out}$$

$$U^e_{out} \sim \frac{d_e V_{Ae}}{\delta_e}$$

$$U^e_{in} \sim \frac{\cancel{\delta_e}}{D_e} \frac{d_e V_{Ae}}{\cancel{\delta_e}} \sim \frac{d_e V_{Ae}}{D_e}$$

cancel

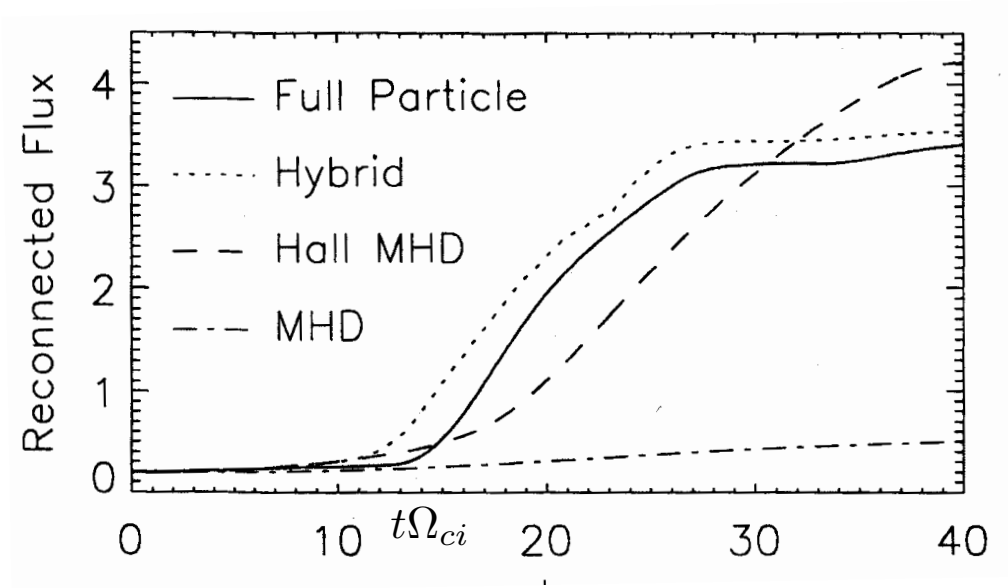
Ion Diffusion Region

$$D_i U_{in} = \delta_i U_{out}$$

$$U_{out} \leq V_{Ai}$$

$$\frac{U_{in}}{V_{Ai}} = \frac{\delta_i}{D_i}$$

GEM Challenge



Birn, et al. 2001
Shay, et al. 2001
Pritchet, 2001
Hesse et al. 2001
Ma & Bhattacharjee, 2001
Otto, 2001

"Universal" Fast Reconnection Rate

$$\frac{\mathbf{U}_{in}}{V_A} \sim 0.1$$

Shay & Drake 1998
Rogers et al 2001
Shay et al 1999
Shay et al 2004
Huba & Rudakov 2004

Claim result is independent of

1. Electron physics
2. Most plasma parameters
3. System size

Kinetic Simulation

Maxwell

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \qquad \nabla \cdot \vec{E} = 4\pi\rho$$

Vlasov

$$\frac{\partial f_s}{\partial t} + \vec{v} \cdot \frac{\partial f_s}{\partial \vec{x}} + \frac{q_s}{m_s}\left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c}\right) \cdot \frac{\partial f_s}{\partial \vec{v}} = 0$$

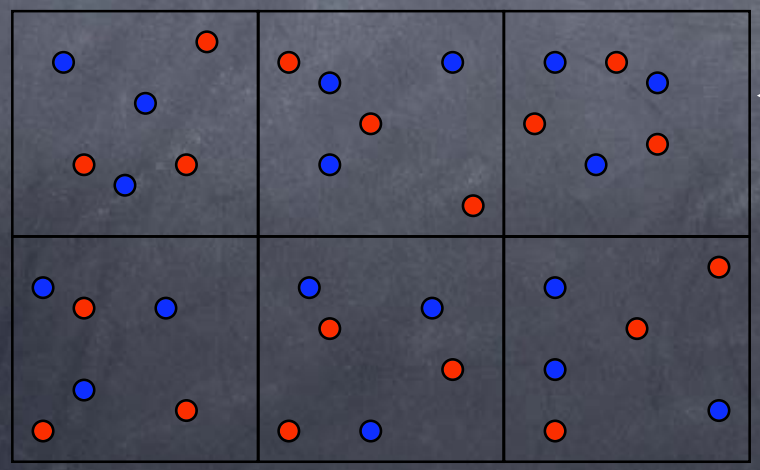
- Coupled by first 2 moments $\rho = \sum_s q_s \int f_s d\vec{v} \qquad \vec{J} = \sum_s q_s \int \vec{v} f_s d\vec{v}$
- Complete description of collisionless plasma
- Possible to add collisions (difficult to do rigorously)
- Easy to solve Maxwell's equations $\vec{A}$ and ϕ
- Vlasov is more difficult - Vlasov code vs PIC Code

PIC = Particle-in-cell

- Introduce "super-particles" - Lagrangian tracers
- Create spatial grid (cells)
- Interpolate position and velocity of particle onto grid $\longrightarrow \rho \quad \vec{J}$
- Compute resulting E and B fields
- Push particles using these self-consistent fields
- Evolution of this system obeys a kinetic equation

Electron

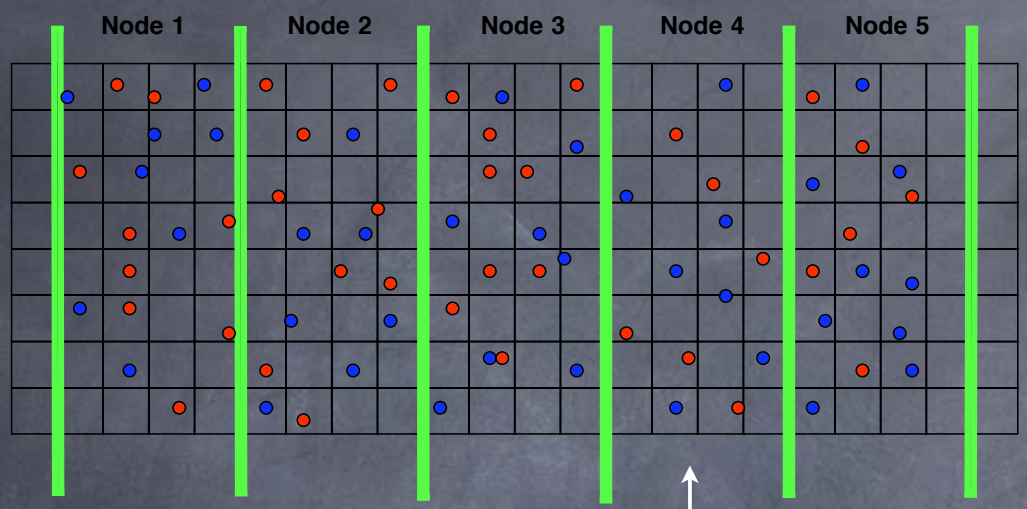
Ion



$\longleftarrow$ N particles/cell

For large N, kinetic equation approaches Vlasov

Parallel Kinetic Simulation



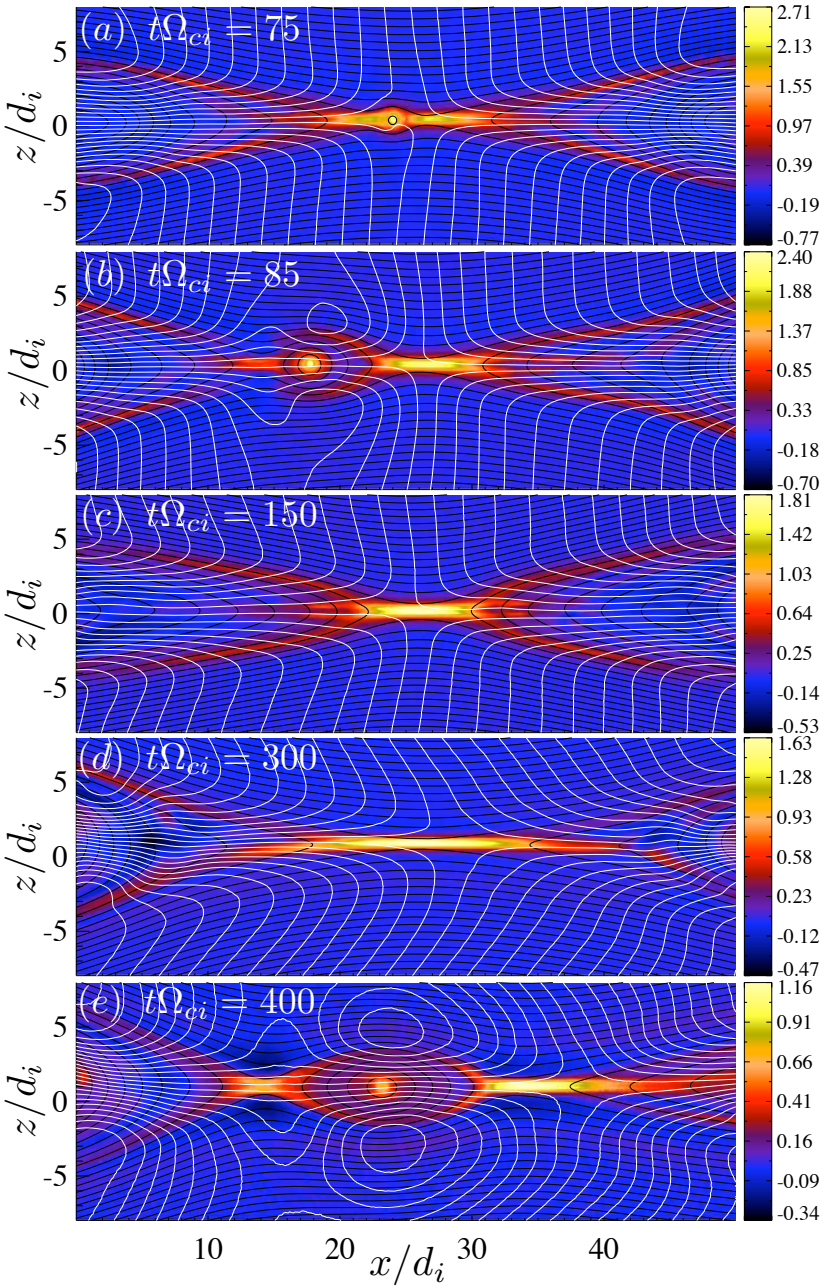
64 processor
Linux Cluster

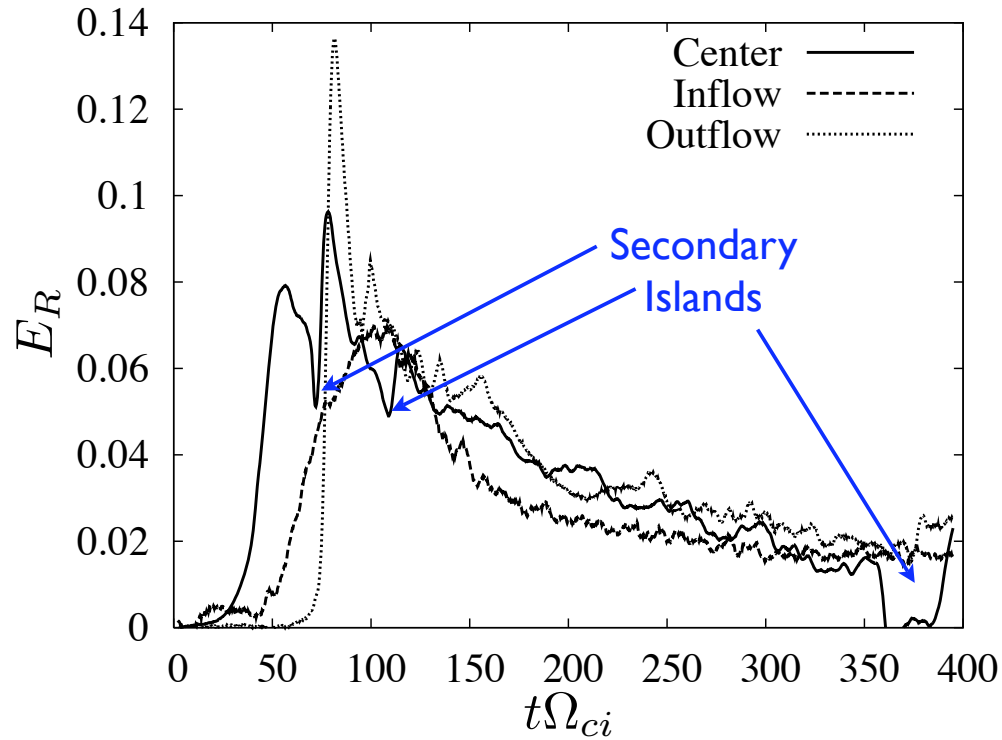
33 - SMP Nodes
2 Opteron CPU's in each
5 GB RAM - 165 GB Total
4 TB Raid Storage
Gigabit Ethernet

Pitfalls of PIC Simulation

- Artificial simulation parameters $\longrightarrow \dfrac{m_i}{m_e} \qquad \dfrac{\omega_{pe}}{\Omega_{ce}}$
- Simulations are mostly 2D
- Small system size
- Short simulation time
- **Periodic boundary conditions**

Large Open Boundary Case





Maximum length of electron diffusion region is limited by secondary island production

Future Work

- Essential to understand how these results scale with mass ratio and plasma parameters
- Evolution in 3D may be dramatically different due to large variety of possible plasma instabilities
- Unlikely to solve the problem with "brute force" computing alone - need theory, space observations, laboratory experiments
- To fully understand the physics of collisionless reconnection remains an enormous challlenge