Measurement of Global Instability of Compact Torus by use of 3-D Tomography

E. Kawamori, A.L. Balandin, K. Urayama,
T. Iwama, T. Ogawa, T. Sumikawa, Y. Ono

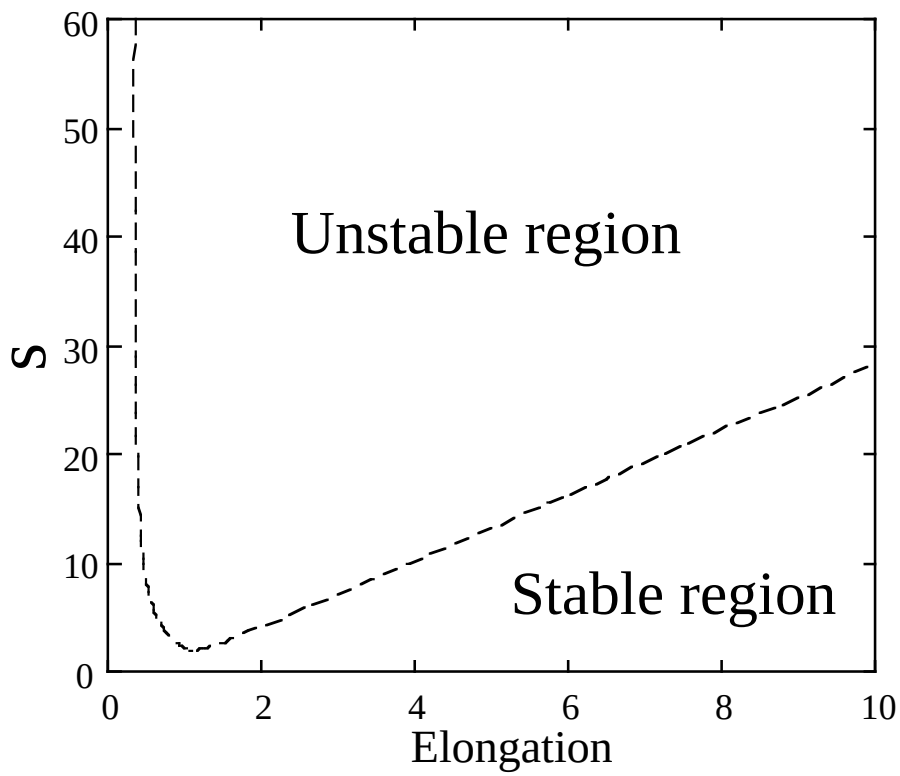
High Temperature Plasma Center, University of Tokyo

need to investigate stability region of tilt mode in a wide range of S-E

- Elongation E and s-value determine the stable region.

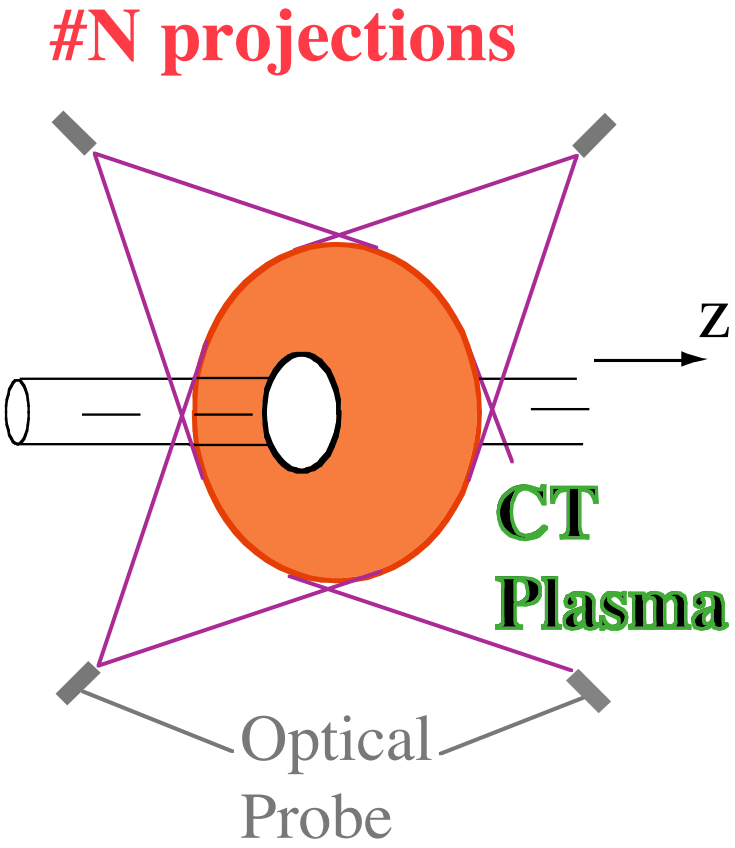
E : ratio of separatrix length to separatrix diameter

s : ratio of separatrix radius to ion gyro-radius



H. Ji, et al., Phys. Plasmas **5** No. 10 3685 (1998).

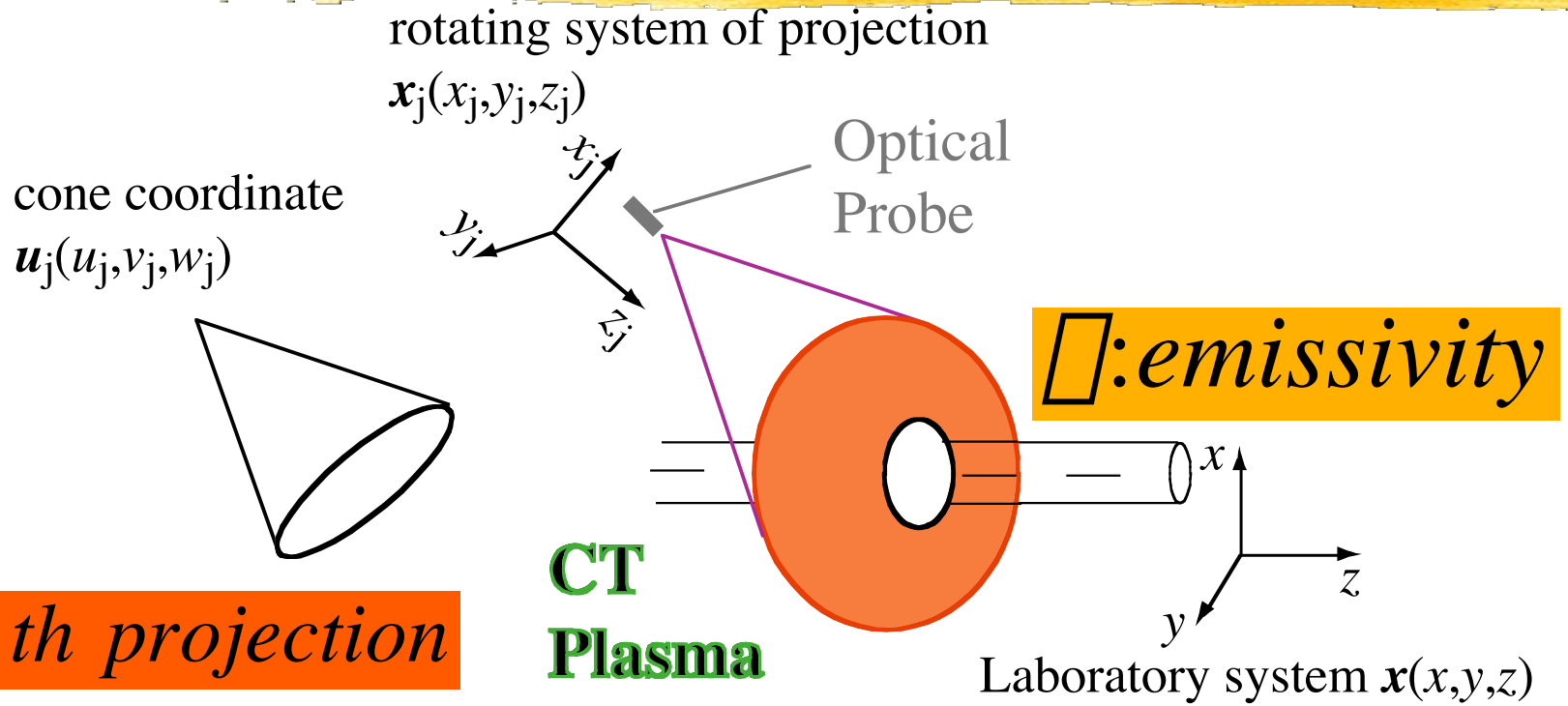
What is the 3-D tomography?



- 3-D emissivity profiles of CTs are directly reconstructed using a number of 2-D projections.
- Our reconstruction error calculation suggests that the most effective number of projection is 16.
- Its reconstruction algorithm is based on the maximum entropy principle.

Principle of 3-D tomography

Transformation of coordinate (1/2)



I_j : jth projection

$$\boldsymbol{X}_j = R_j \boldsymbol{X}$$

$$R_j(\theta,\phi) = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{bmatrix}$$

Transformation of coordinate (2/2)

$$u_j = \frac{x_j}{1 + z_j / d_j}, \qquad x_j = u_j (1 + w_j / d_j)$$

$$v_j = \frac{y_j}{1 + z_j / d_j}, \qquad y_j = v_j (1 + w_j / d_j)$$

$$w_j = z_j, \qquad z_j = w_j .$$

projection I_j, emissivity ρ

$$I_j(u, v) = \int_{R^2} dX \rho(\boldsymbol{x}) \delta(u - U_j(\boldsymbol{x})) \delta(v - V_j(\boldsymbol{x}))$$

$$= \int_{L_j} dw \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| \rho(x, y, z)$$

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = (1 + w_j / d_j)^2$$

Using Maximum Entropy Method (MEM) (1/2)

Entoropy functional : an index of smoothness of an image

$$\Phi(\rho) = -\int_D dw\, \rho(\mathbf{x}) \ln(\rho(\mathbf{x}) V)$$

V : normalization constant

Maximize

Lagrange functional with $\tilde{I}_j = const.$

$$L(\rho, \lambda) = \Phi(\rho) - \sum_{j=1}^{J} du\, dv\, \lambda_j(u,v) \tilde{I}_j(u,v)$$

λ : Lagrange multiplier

J : number of projections

$$\tilde{I}_j(u,v) \equiv I_j(u,v) - \int dw\, |J| \rho(x_j, y_j, z_j)$$

Using Maximum Entropy Method (MEM) (2/2)

From $\nabla L = 0$

$$\rho(\mathbf{x}) = \frac{1}{V} \prod_{l=1}^{J} H_l(U_j(\mathbf{x}), V_j(\mathbf{x}))$$

Calculating $H_l(U_j(\mathbf{x}), V_j(\mathbf{x}))$

results in determination of $\rho(\mathbf{x})$

$$I_j(u,v) = \frac{1}{V} \int_{\Omega_j} dw \, |\mathcal{J}| \prod_{l=1}^{J} H_l(U_j, V_j)$$

$$I_j(u,v) = \frac{1}{V} H(u,v) \int_{\Omega} dw \, |\mathcal{J}| \prod_{l \neq j}^{J} H_l(U_{lj}, V_{lj})$$

H_j are obtained by iteration of the calculation

Experimental data

$$H_j^{i+1}(u,v) = \frac{I_j(u,v)V}{\int_{l_j} dw \left|1 + w/d_j\right|^2 \sum_{l=1}^{J} H_l(U_{lj},V_{lj})}$$

for $j = i \pmod J + 1$:

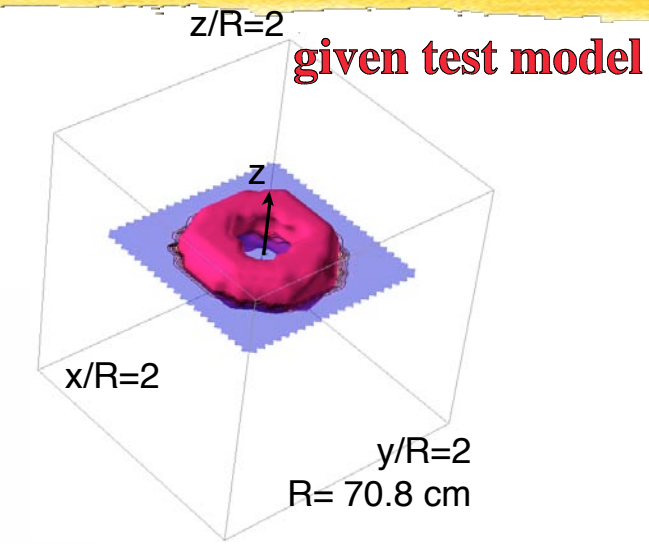
$$H_j^{i+1}(u,v) = H_j^i(u,v) \quad \text{for} \quad j \neq i \pmod J + 1:$$

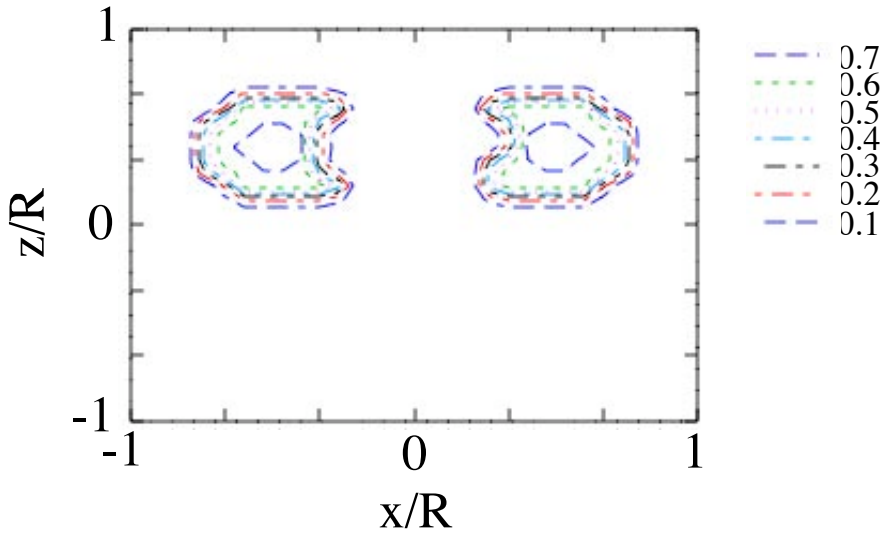
$$H_j^0(u,v) = \begin{cases} 1, \text{ if } I_j(u,v) \neq 0 \\ 0, \text{ if } I_j(u,v) = 0 \end{cases}$$

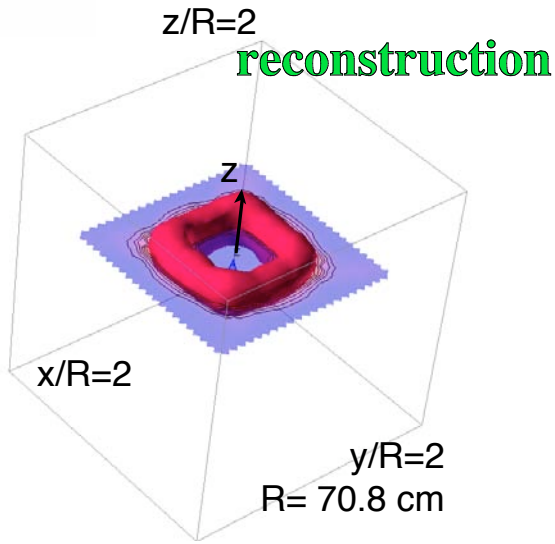
Initial value

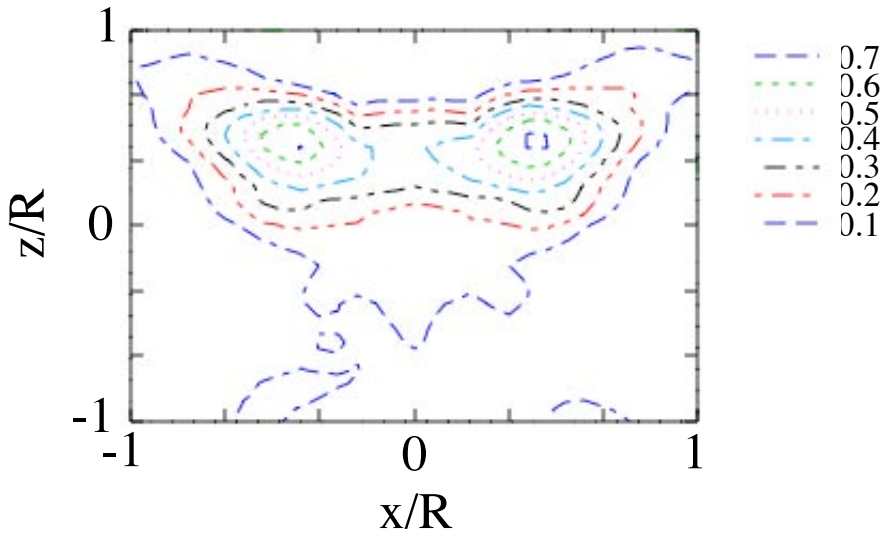
Calculation of a test model

ch.7
projection

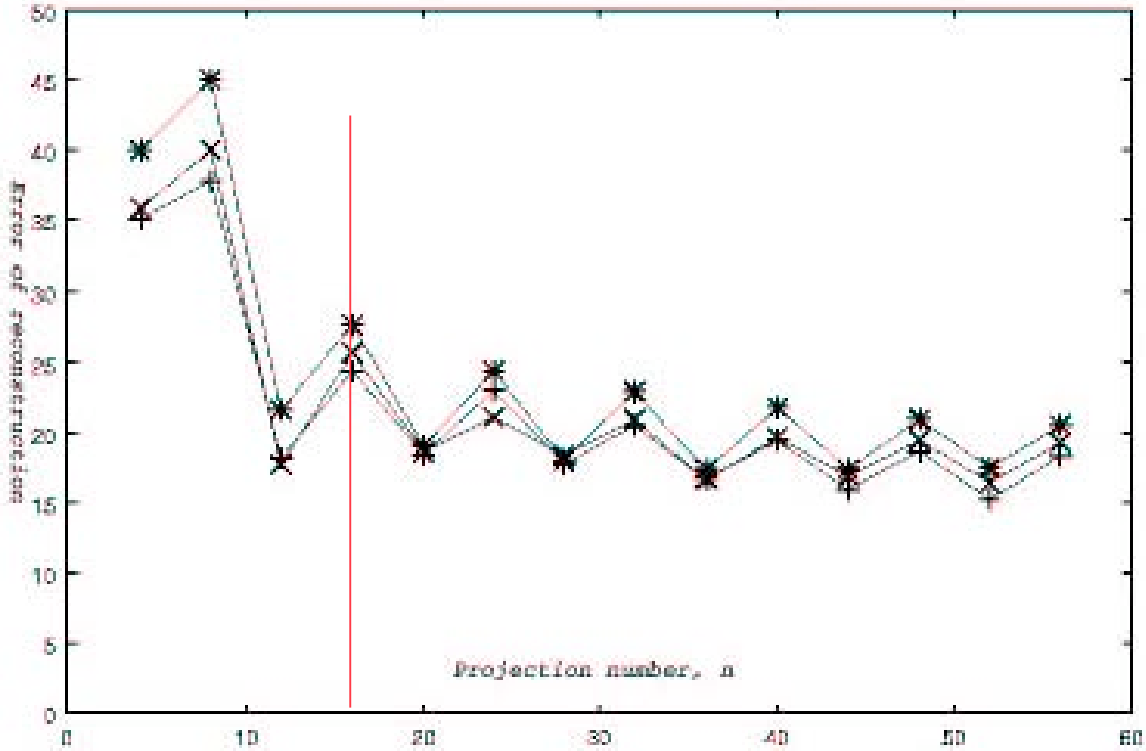








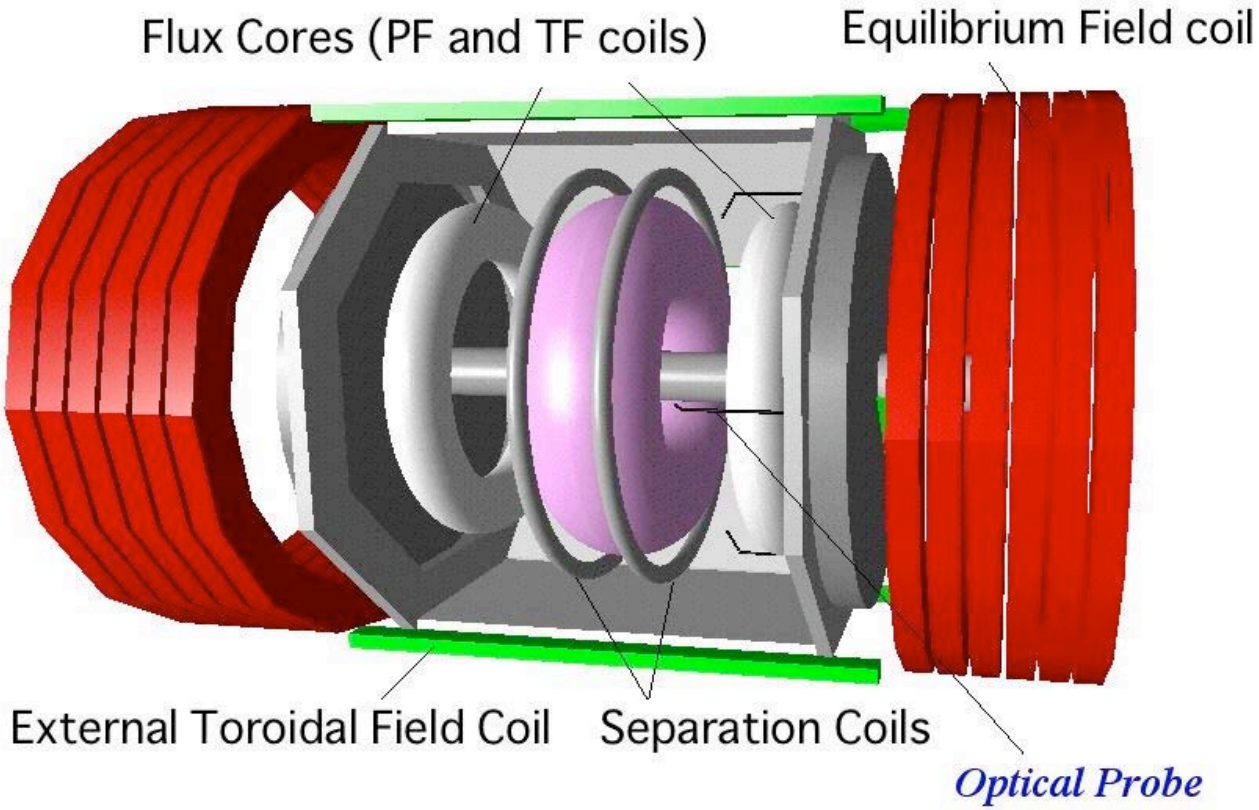
The minimum number of projection is 16 based on our model calculation.

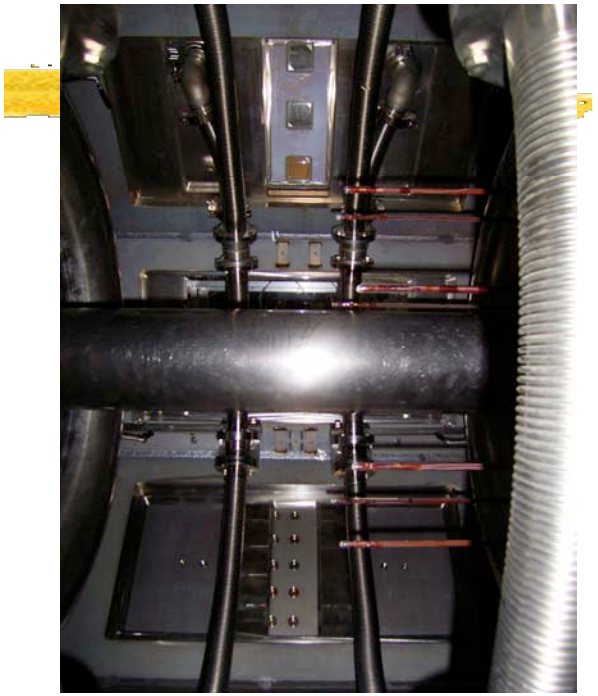


16

Reconstruction error as a function of projection number for toroidal mode n=1 to n=3

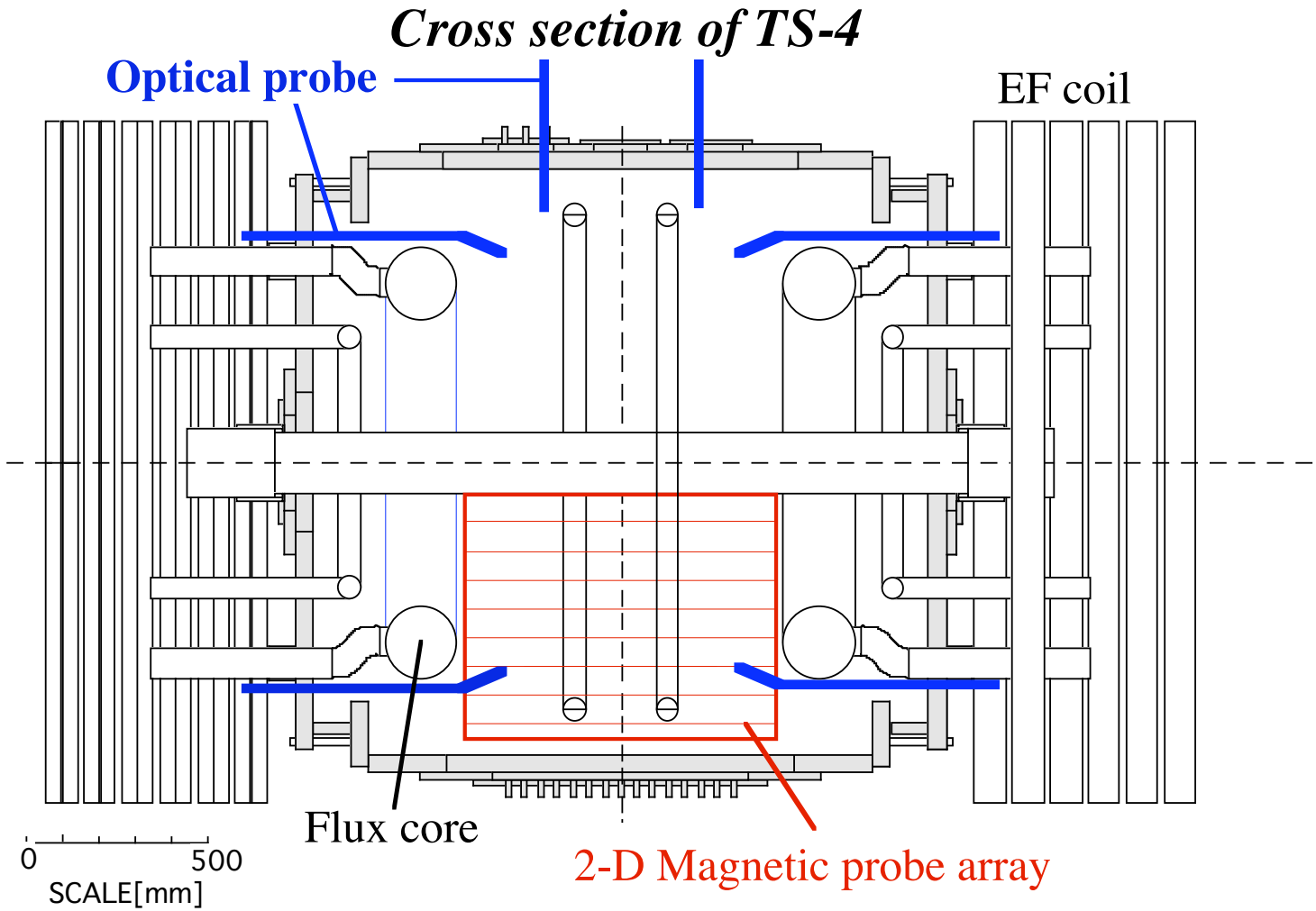
TS-4 Merging/CT Device





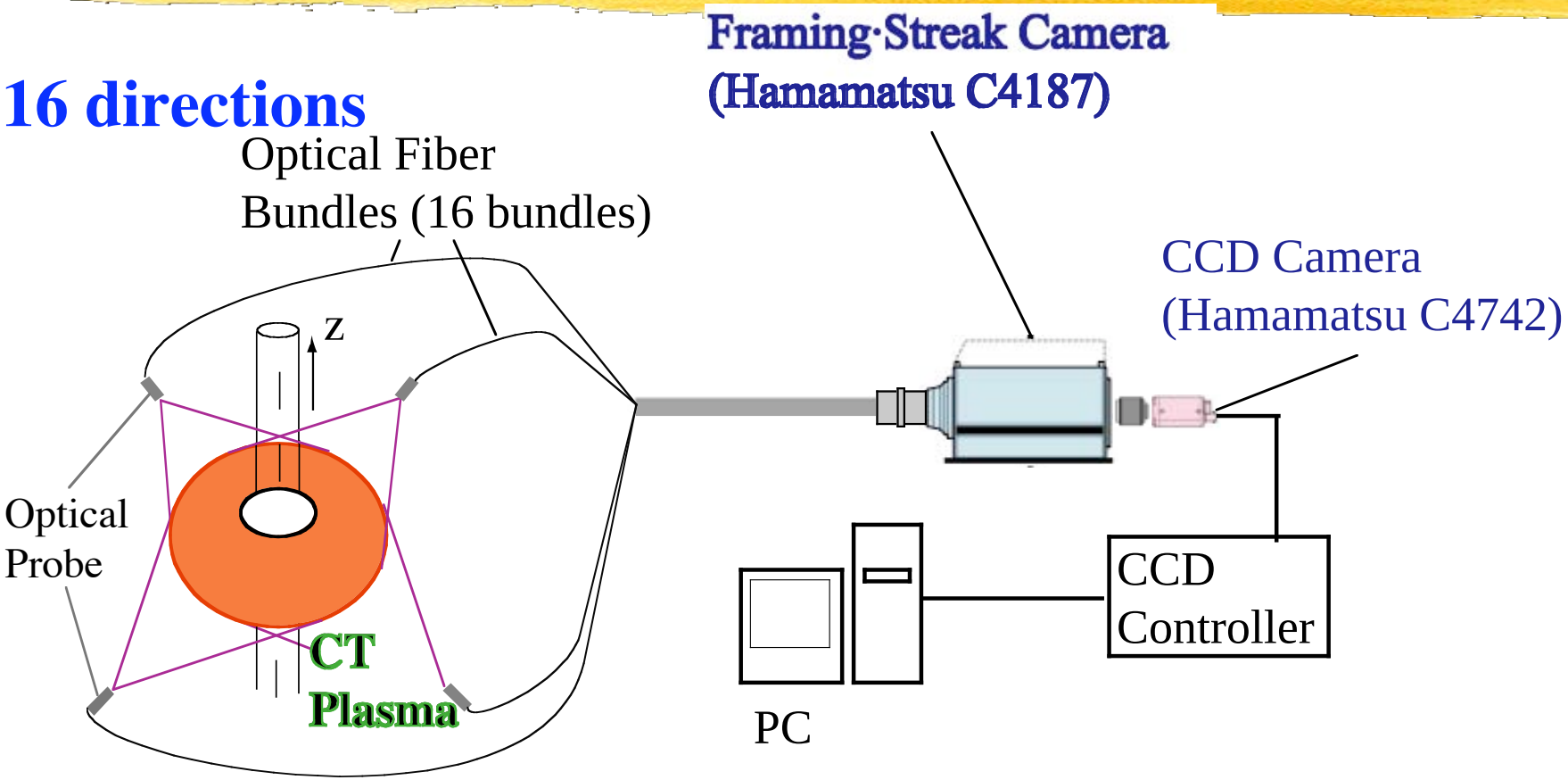
- Utilizes fluxcores with PF and TF coils for poloidal and toroidal flux injection.
- All type of CTs (q_o~-0.7-5) can be produced in a single device.
- R=0.4~0.6 m, R/ a~1.5, B_{t0}~0.05 T .

Measurement Systems in TS-4



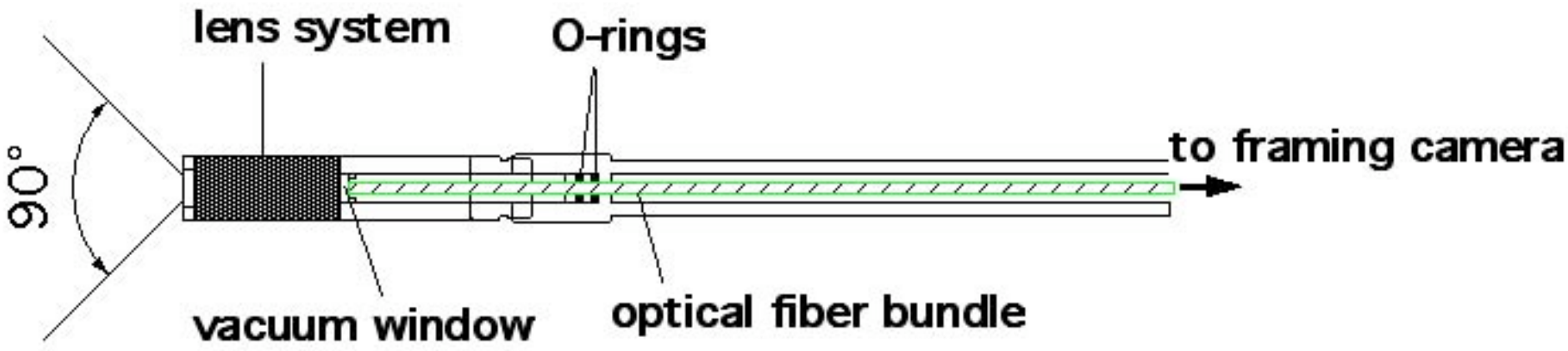
- A 2-D array of magnetic probe for calculation of flux contour
- Another 8 magnetic probes to measure toroidal modes (n=1~4)

3-D Tomography System

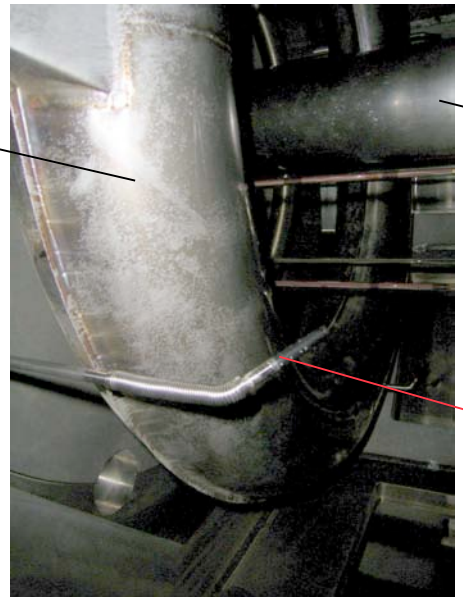


Schematic drawing of the 3-D tomography system

Optical probe



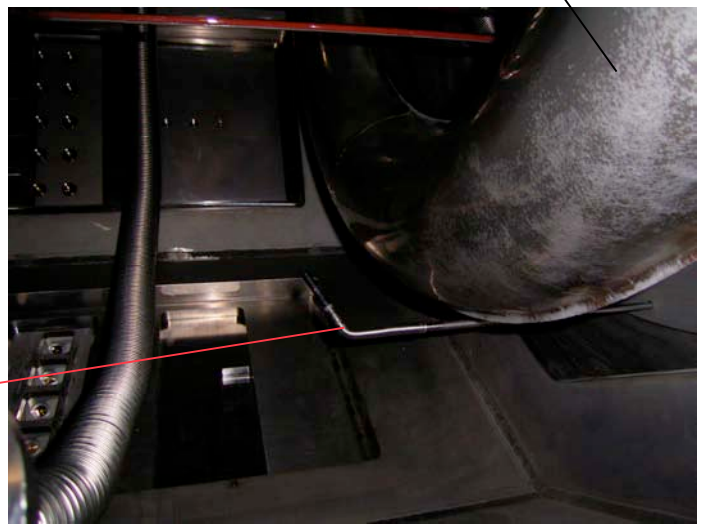
Flux core



Center post

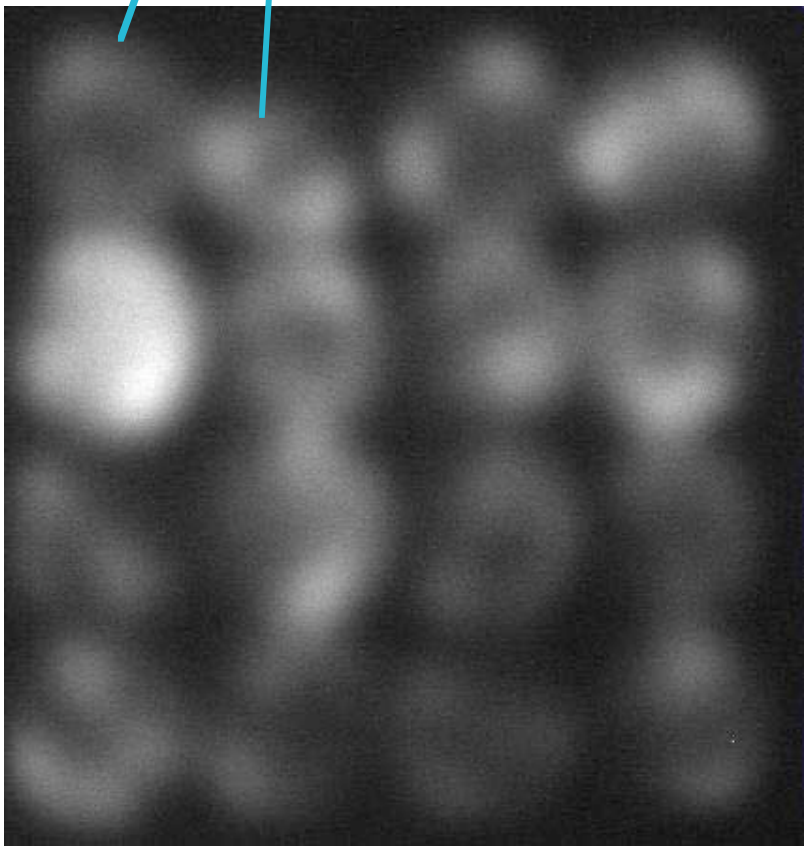
Optical probe

Flux core

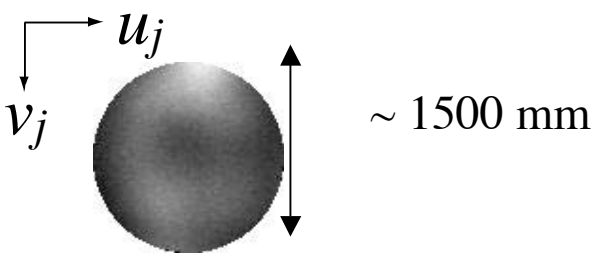


Data of the 16 projections

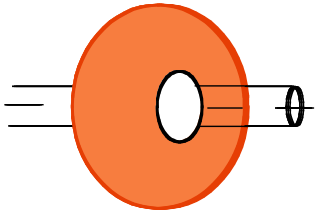
ch 1, ch 2,.....



Example



f4122009-410us
Pos. 4 (ch.9)

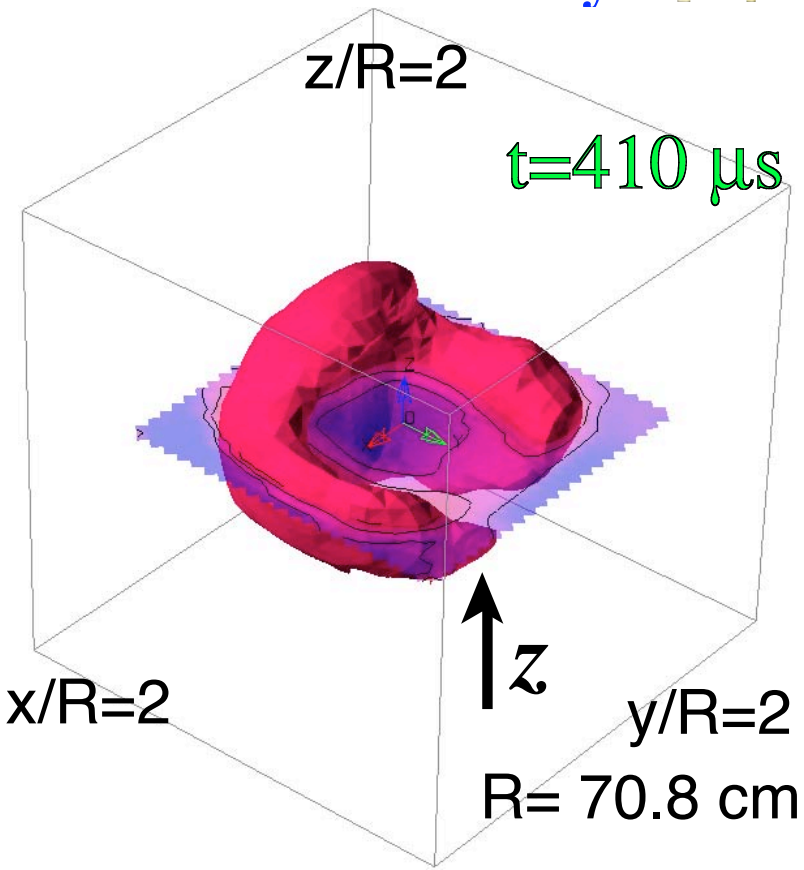


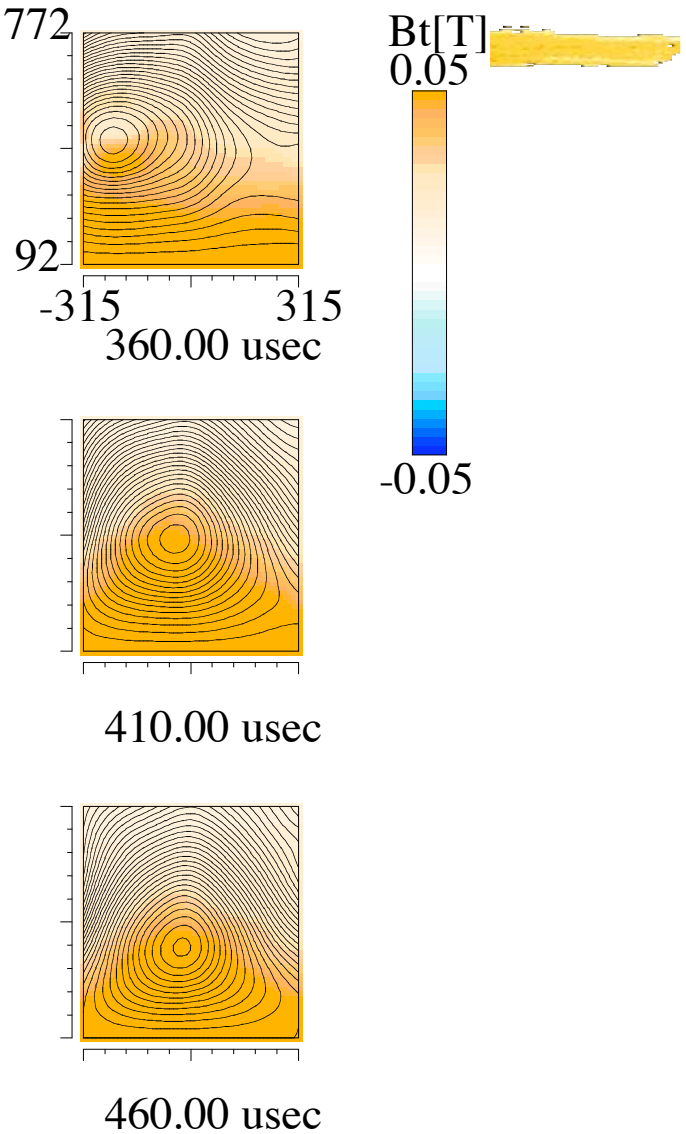
Shadow effect of the center post was seen.

Reconstruction of emissivity in ST

Data Set Name : f4113024

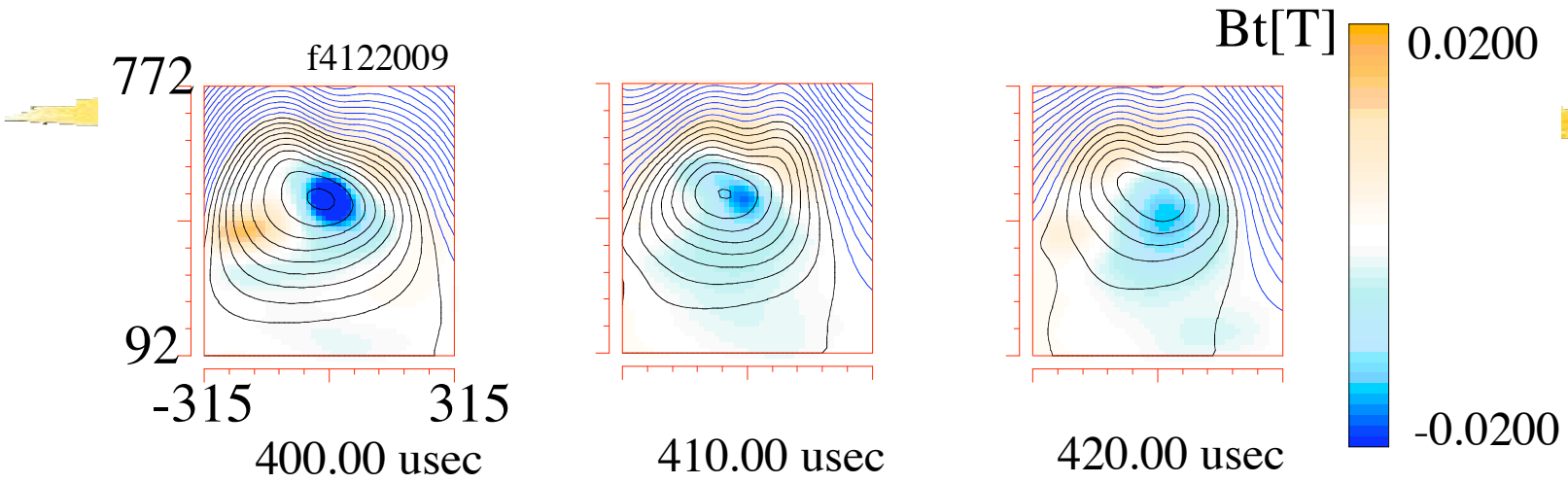
Isosurface of the emissivity

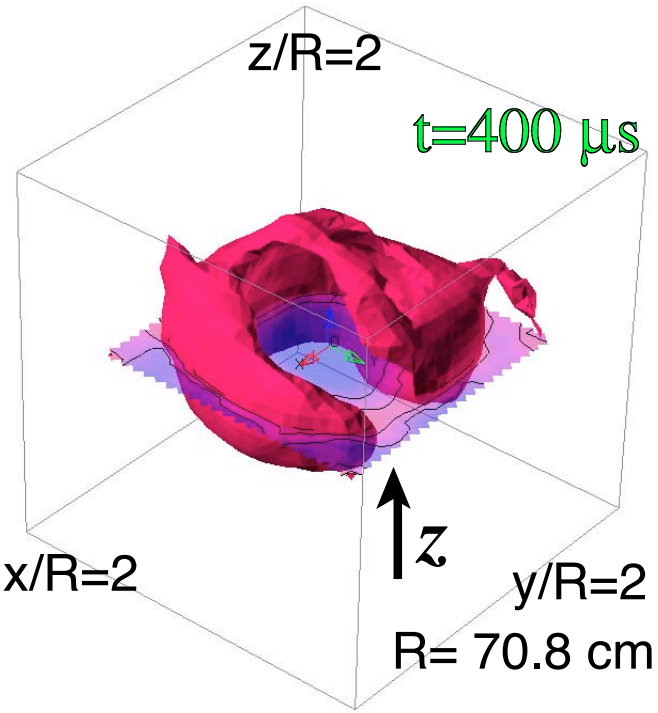


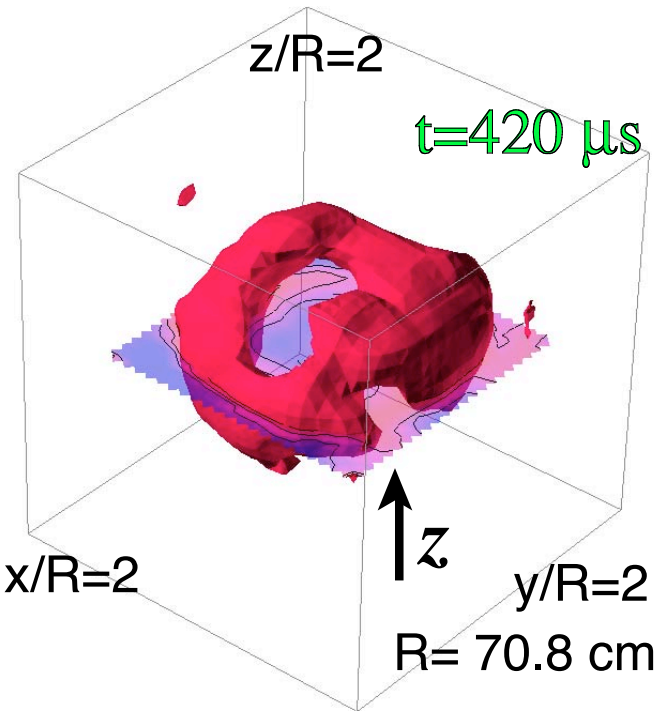


- Torus shape was observed however partially broken.

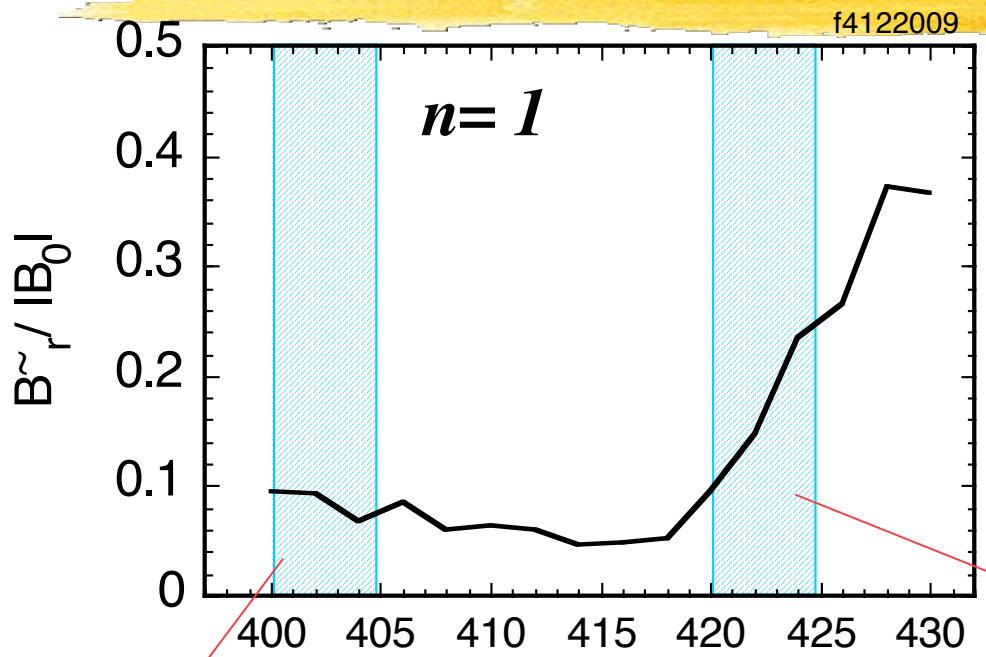
Observed an oblate FRC tilted



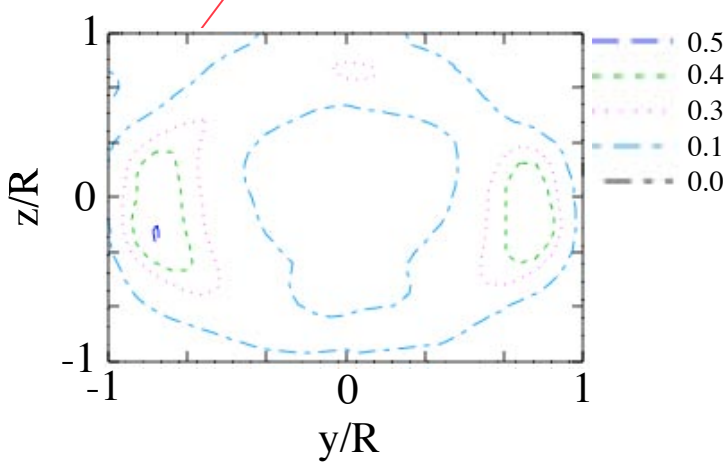


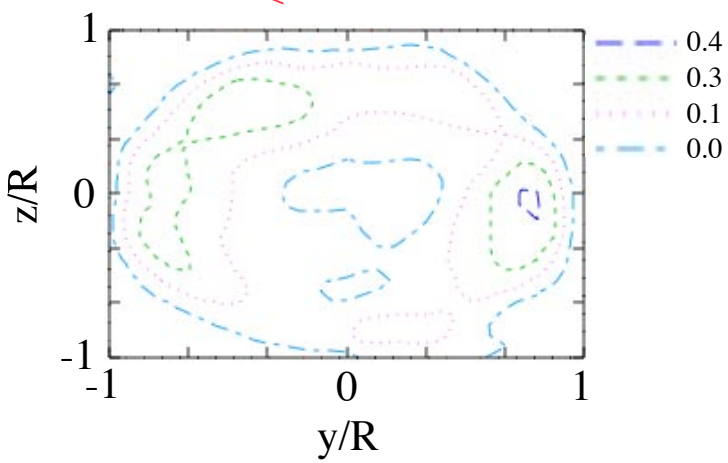


Tilt mode (n=1) was observed by tomography



Contour on y-z plane tilted simultaneously with growth of *n*=1





Summary

- The new 3-D tomography reconstruction was first experimentally tested for a powerful monitor of CT configuration using a 16-projection system of visible light tomography.

- The n=1 tilt motion of FRC and the stable ST were measured successfully using the 16-projections of plasma light emissivity .

- Comparison with magnetic measurements suggests that the reconstruction error caused by edge plasma light are left unsolved.